Welcome to

FUNDAMENTAL MATHEMATICS

FOR INDUSTRY EDUCATION

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9641103 Fundamental Mathematics for Industry Education

Course Description

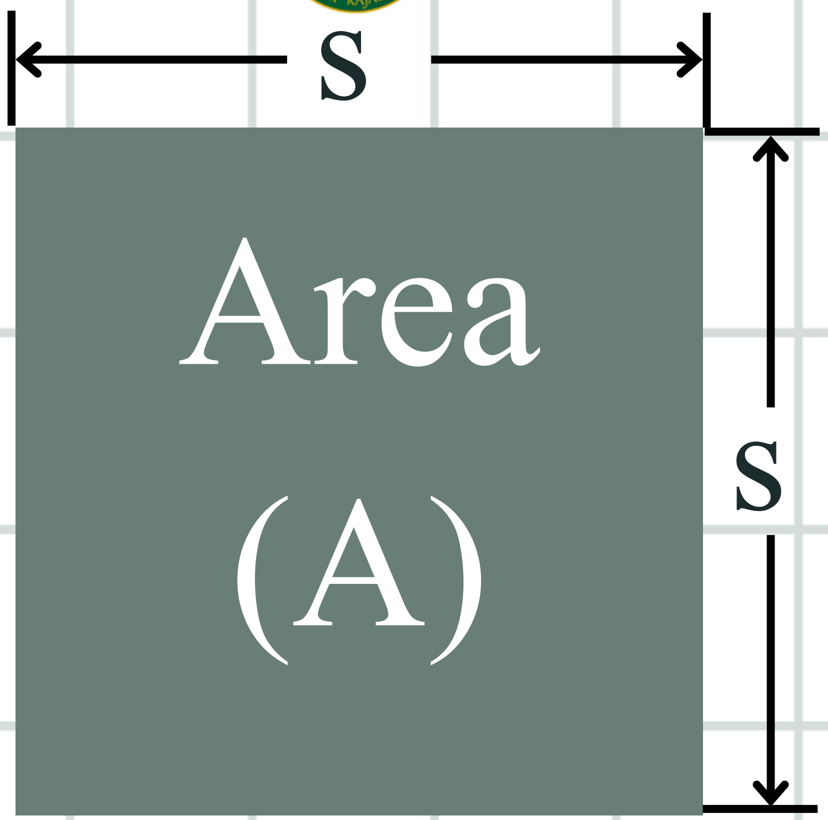
Function, limit and continuity, multivariate equation, matrix, differentiation, integration, and application for Industry education

Lesson 1: Functions

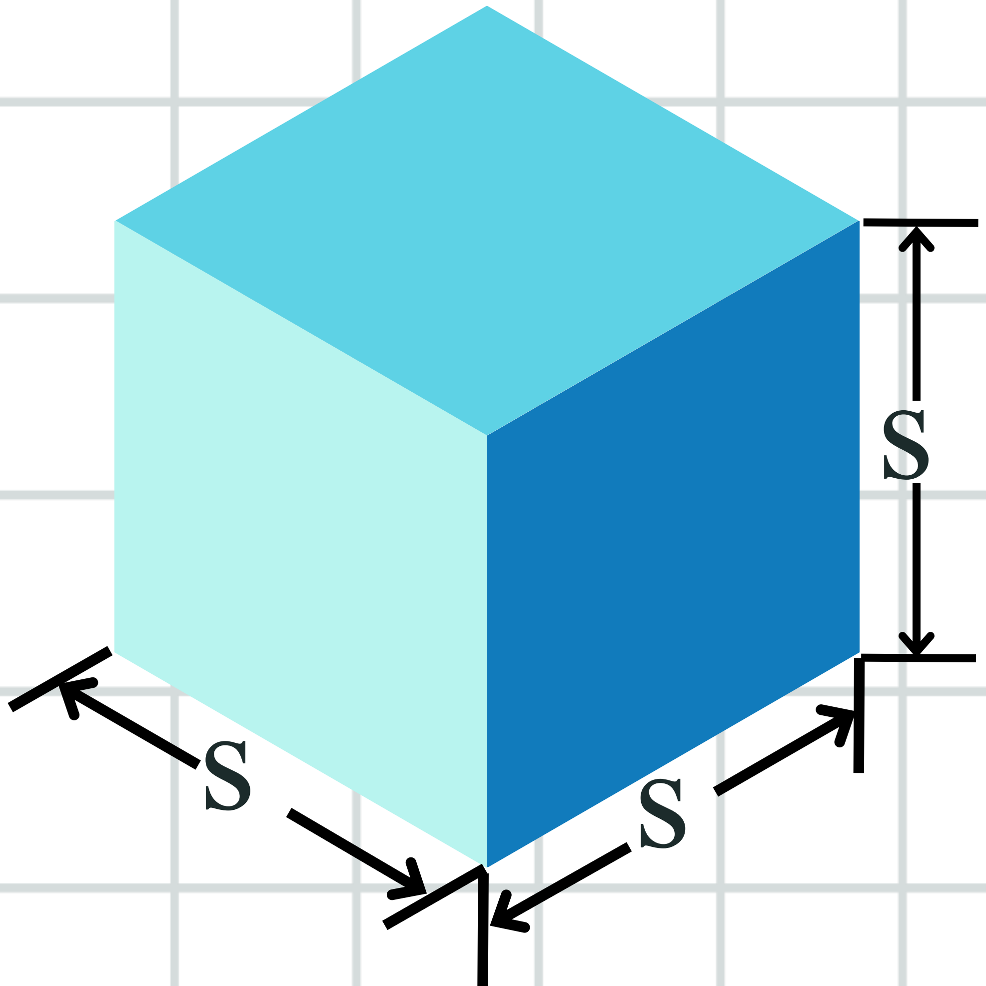
What is Functions?

In this lesson, we explore the relationship between two or more variables that change in correspondence with one another.

For instance, the area of a square depends on the length of its side, and the volume of a cube depends on the length of its edge.



$$A = s^2$$



$$V = s^3$$

Example

A company wants to manufacture a cylindrical can contain one of its liquid products. If the can has a capacity of approximately 22 cubic inches, express the surface area (including the lateral surface, top, and bottom) as a function of the radius, and determine the domain of the function.

Solution

Step 1: Define variables

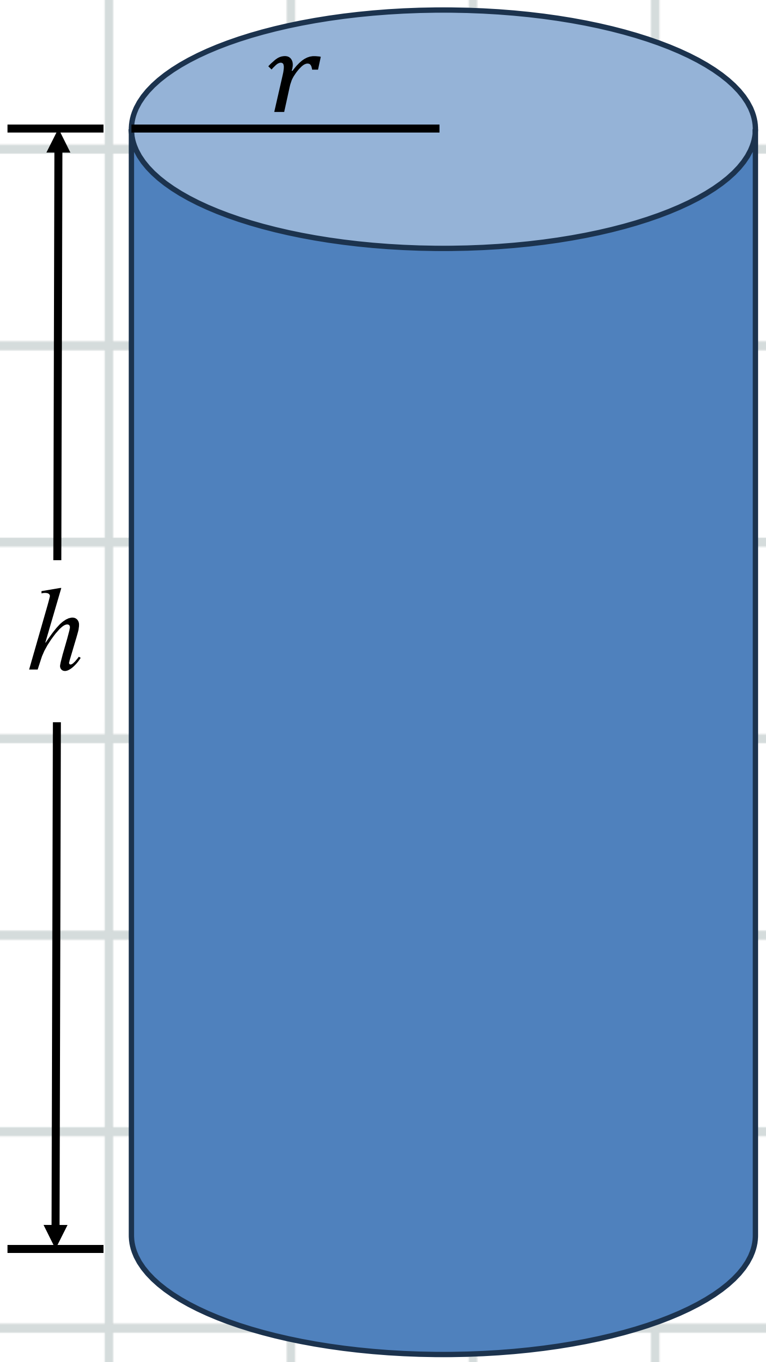
Let

r = radius of the base

h = height of the cylinder

Step 2: Use the volume formula

The volume V of a cylinder is given by $\boxed{V = \pi r^2 h}$



Solution

Since the volume is 22 in^3

$$22 = \pi r^2 h$$

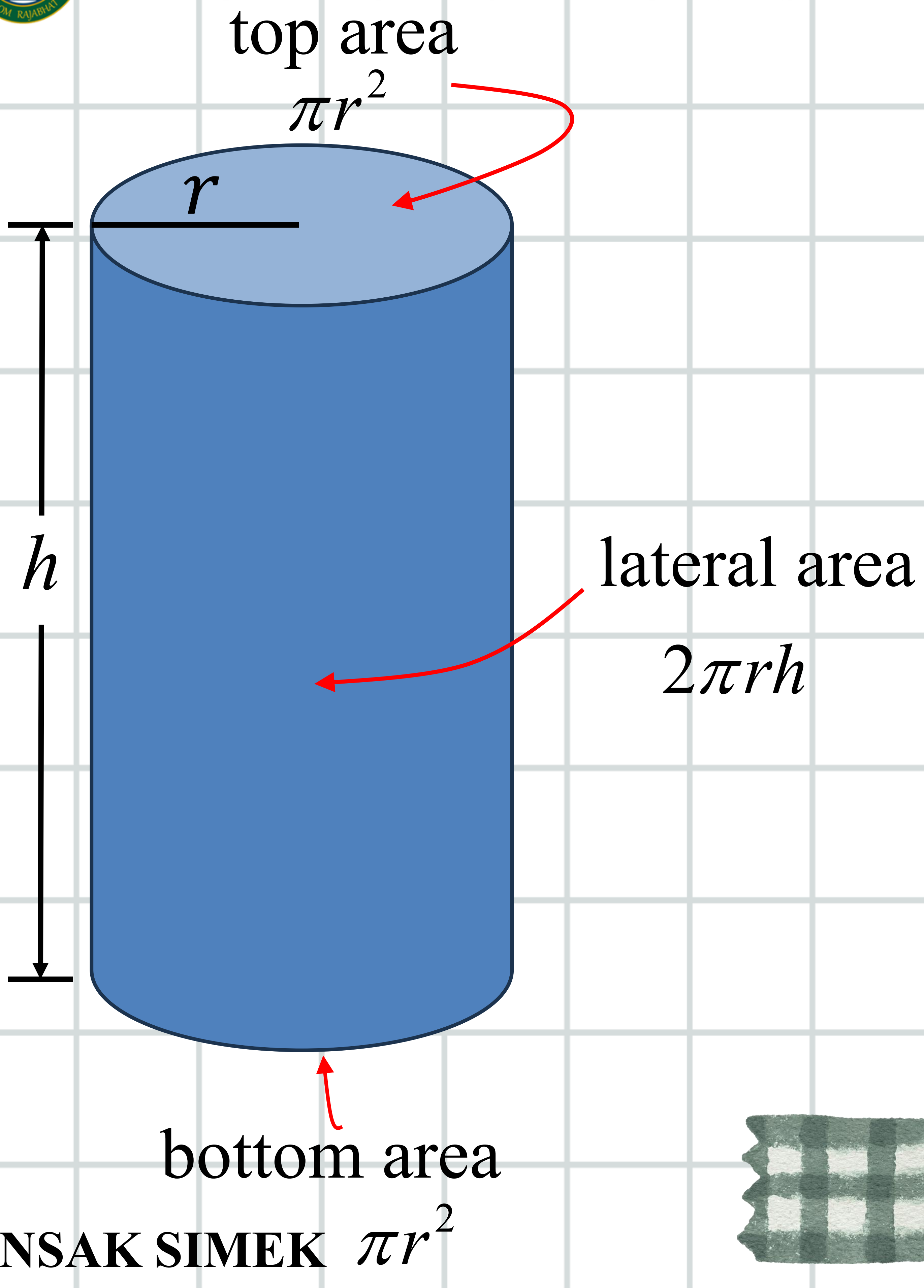
Solve for h

$$h = \frac{22}{\pi r^2}$$

Step 3: Write the surface area formula

The total surface area A of a cylinder includes $A = A_{lateral} + A_{top} + A_{bottom}$

$$\boxed{A = 2\pi r h + 2\pi r^2}$$



Solution

Step 4: Express A in terms of r

Substitute $h=\dfrac{22}{\pi r^2}$ into the equation for

$$A(r)=2\cancel{\pi}\cancel{r}\left(\frac{22}{\cancel{\pi}\cancel{r^2}}\right)+2\pi r^2$$

Simplify:

$$\boxed{A(r)=\frac{44}{r}+2\pi r^2}$$

function of the radius

Step 5 : Determine the domain

Since r represents the radius of the can :

$$r>0$$

it cannot be zero or negative

Therefore, the domain is

$$\boxed{(0,\infty)}$$

the domain of the function

Which of the following represents y as a function of x?

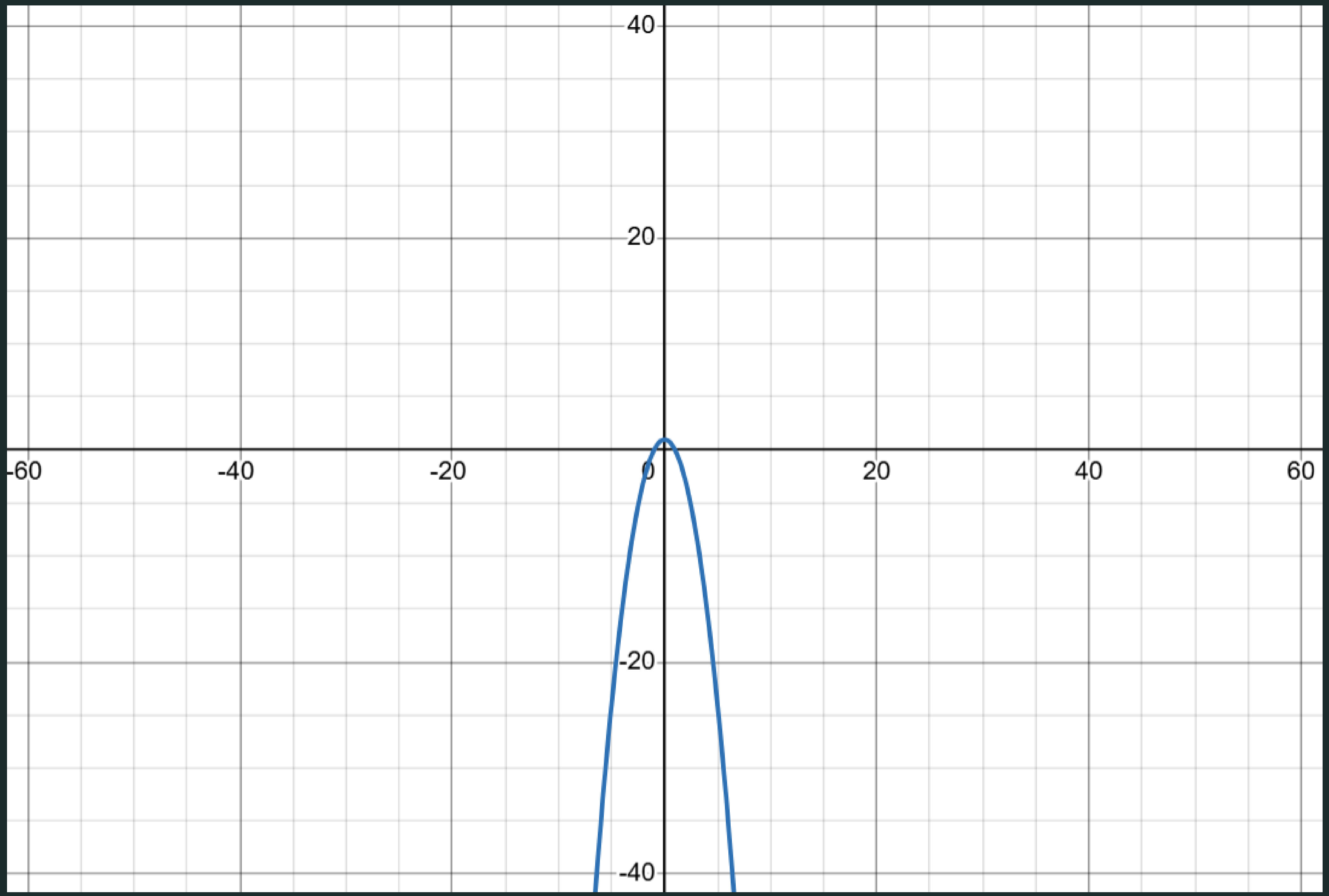
$$x^2 + y = 1$$

$$y^2 + x = 3$$

$$\sqrt{y+1} = \frac{1}{1-x}$$

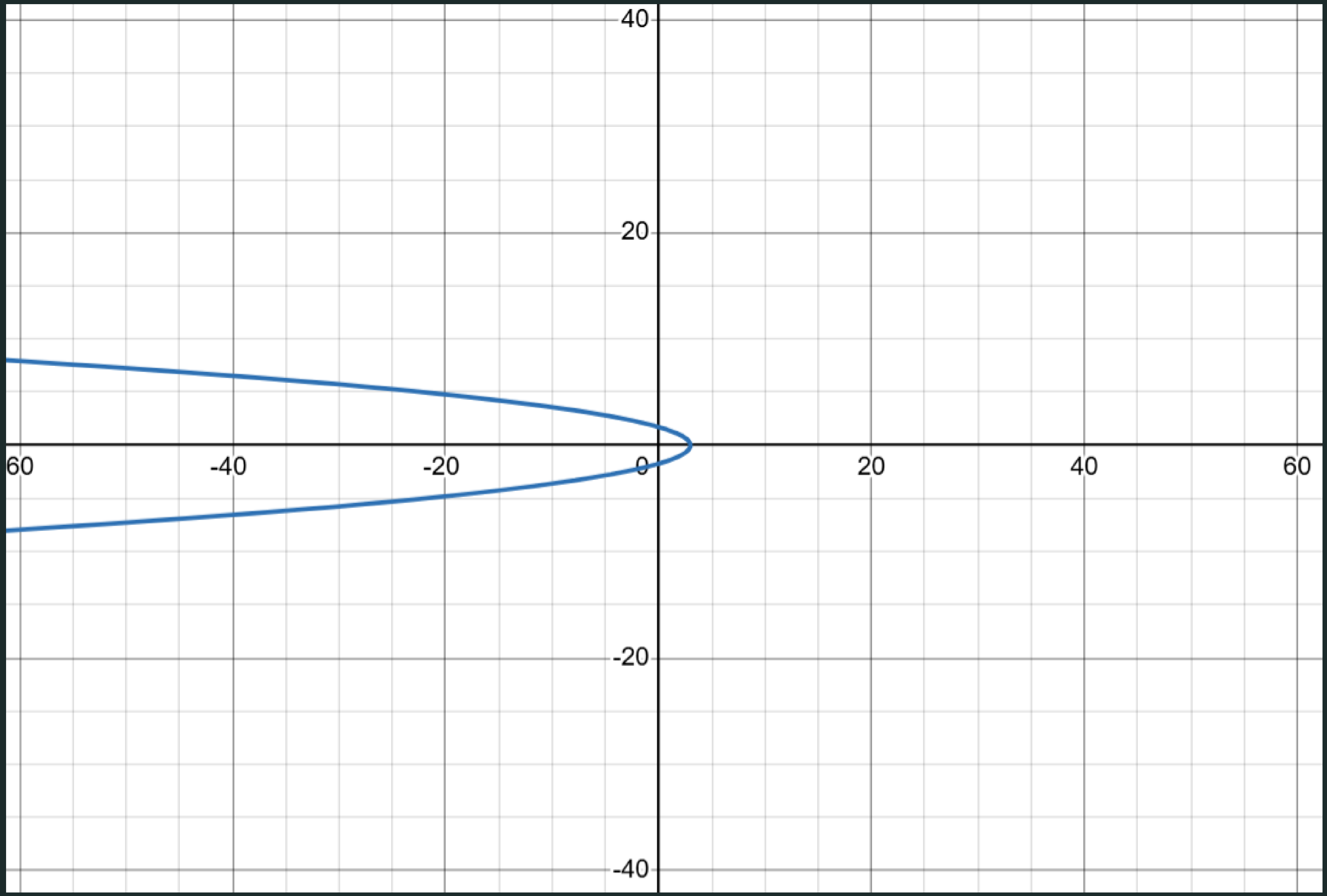
Which of the following represents y as a function of x?

$$x^2 + y = 1$$



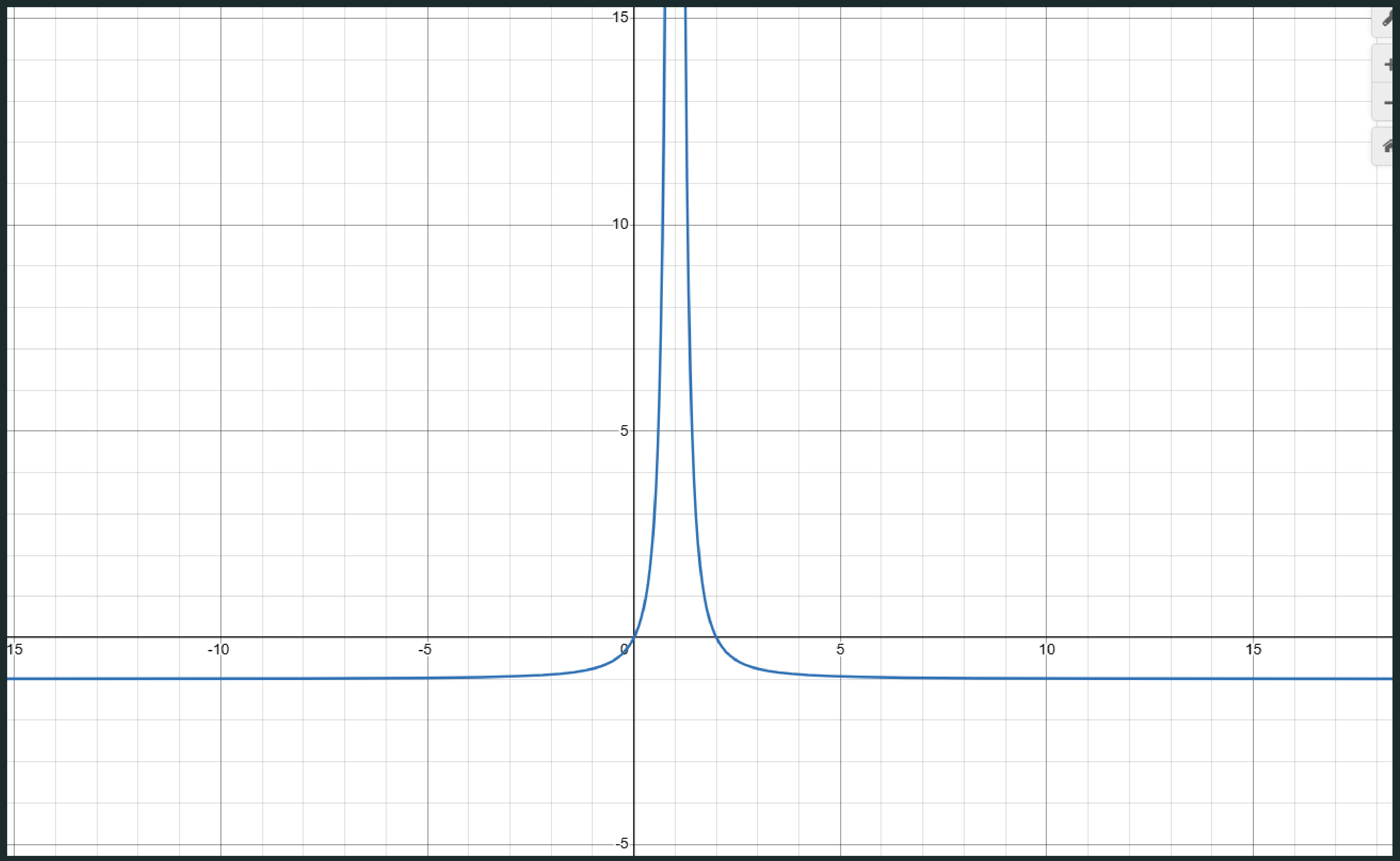
Which of the following represents y as a function of x?

$$y^2 + x = 3$$



Which of the following represents y as a function of x?

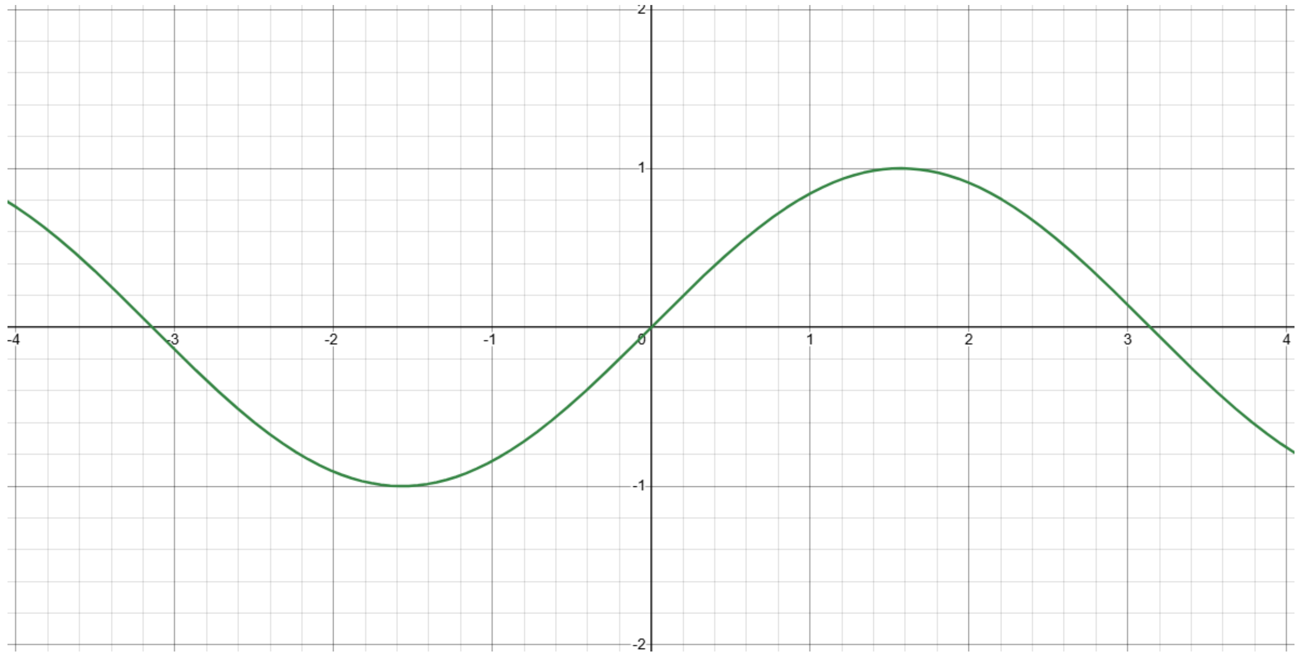
$$\sqrt{y+1}=\frac{1}{1-x}$$

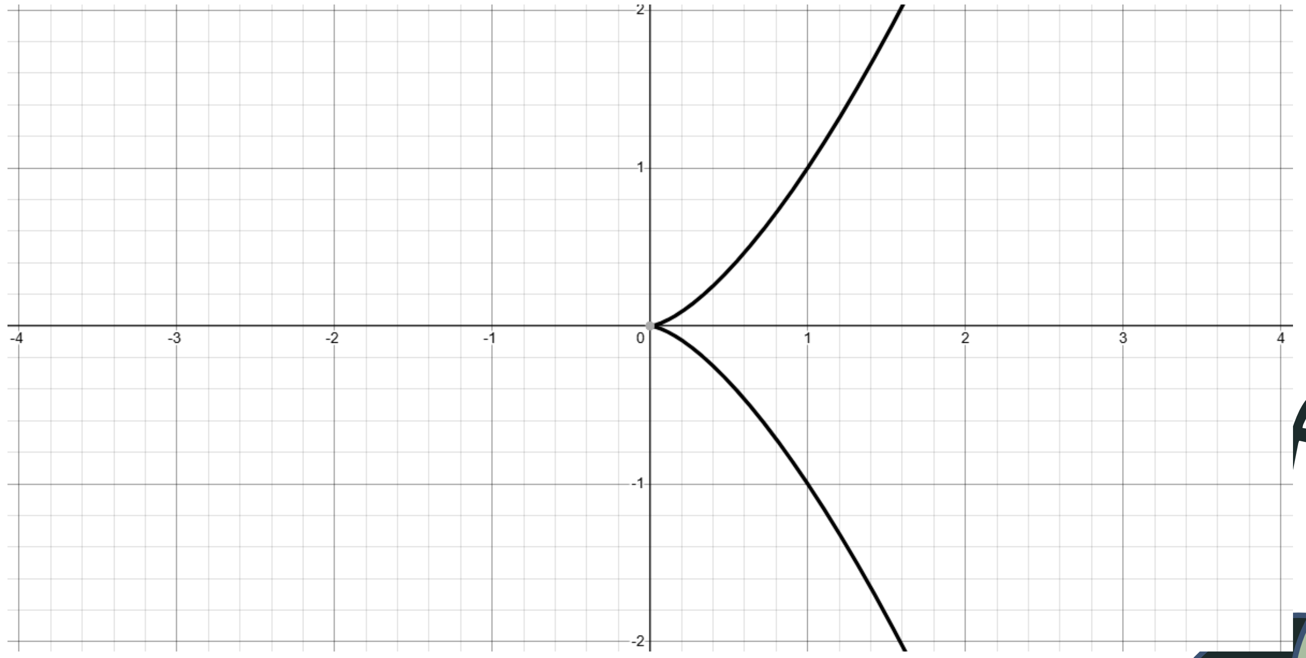


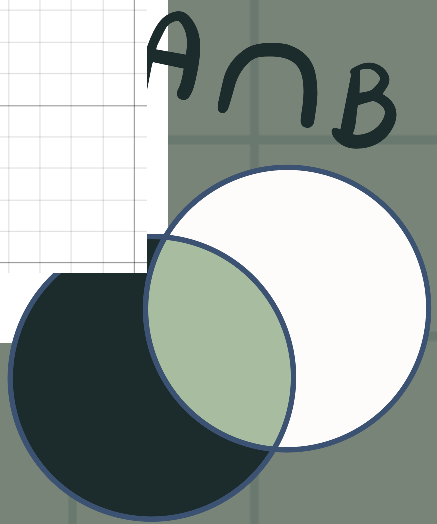
The Vertical Line Test

If every vertical line parallel to the y-axis intersects the graph of a relation between x and y at no more than one point, then the graph of that relation represents a functions.

Conversely, if there exists any vertical line parallel to the y-axis that intersects the graph at two or more points, then the graph does not represent a functions.

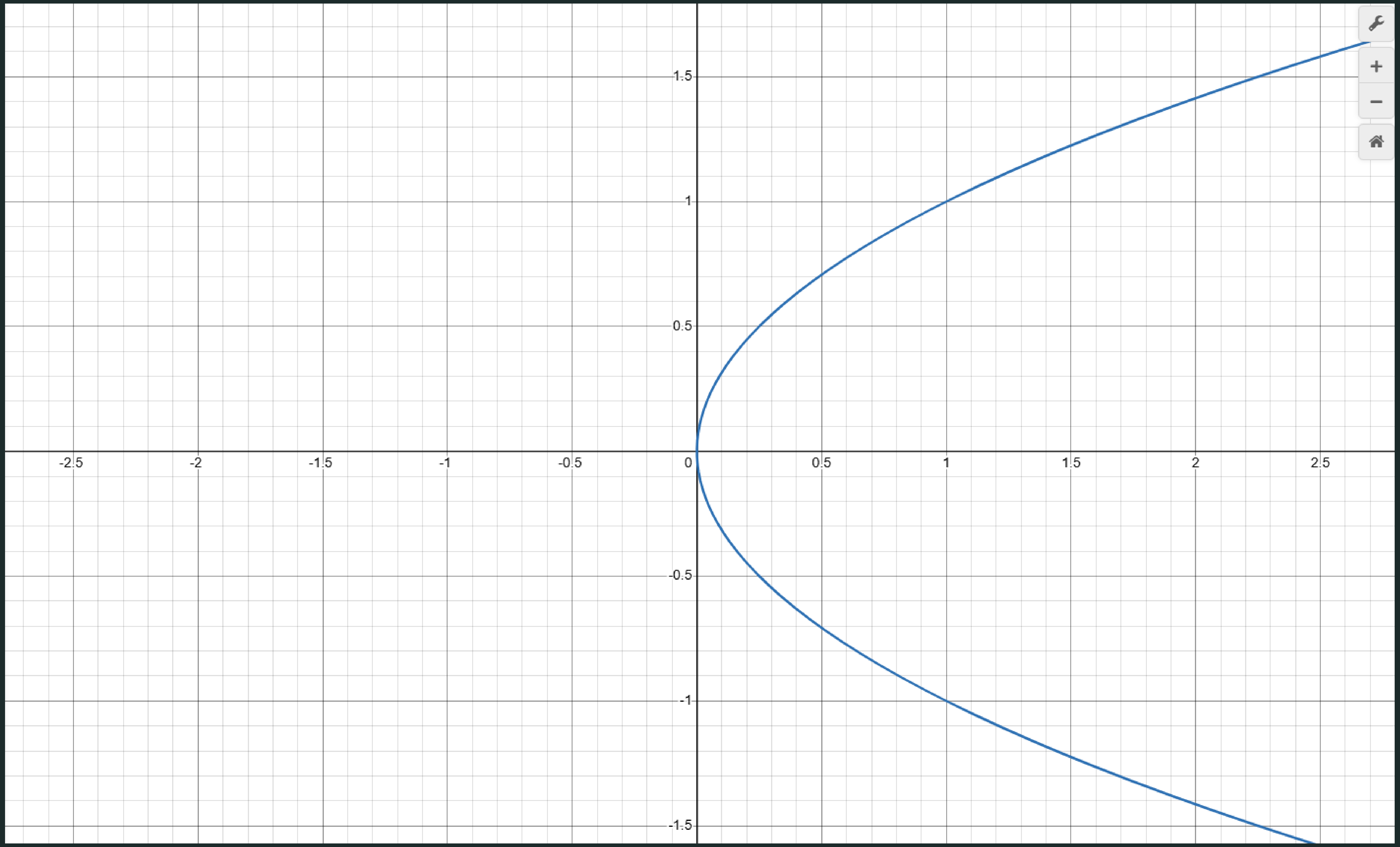






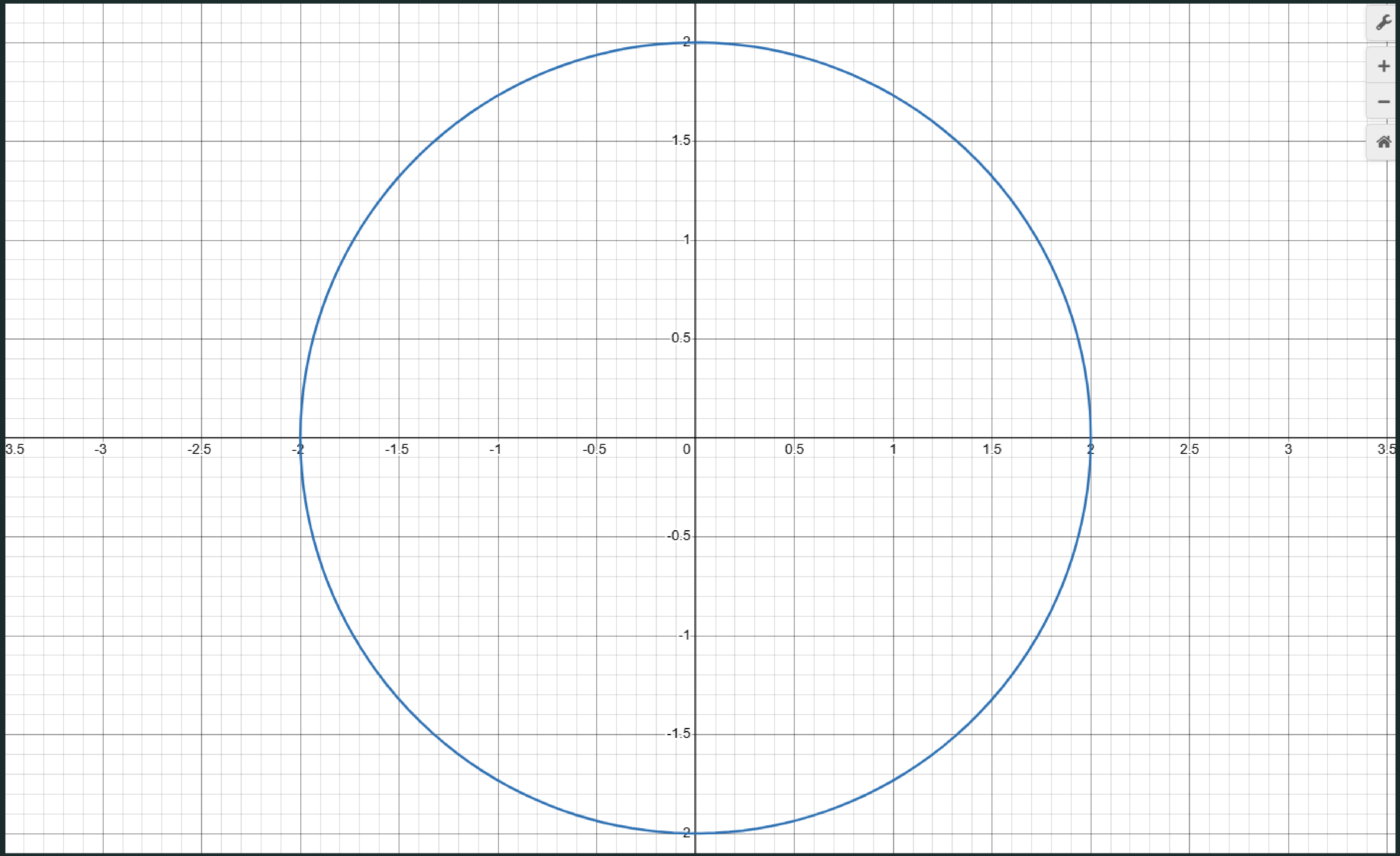
Which of the following represents y as a function of x?

$$x = y^2$$



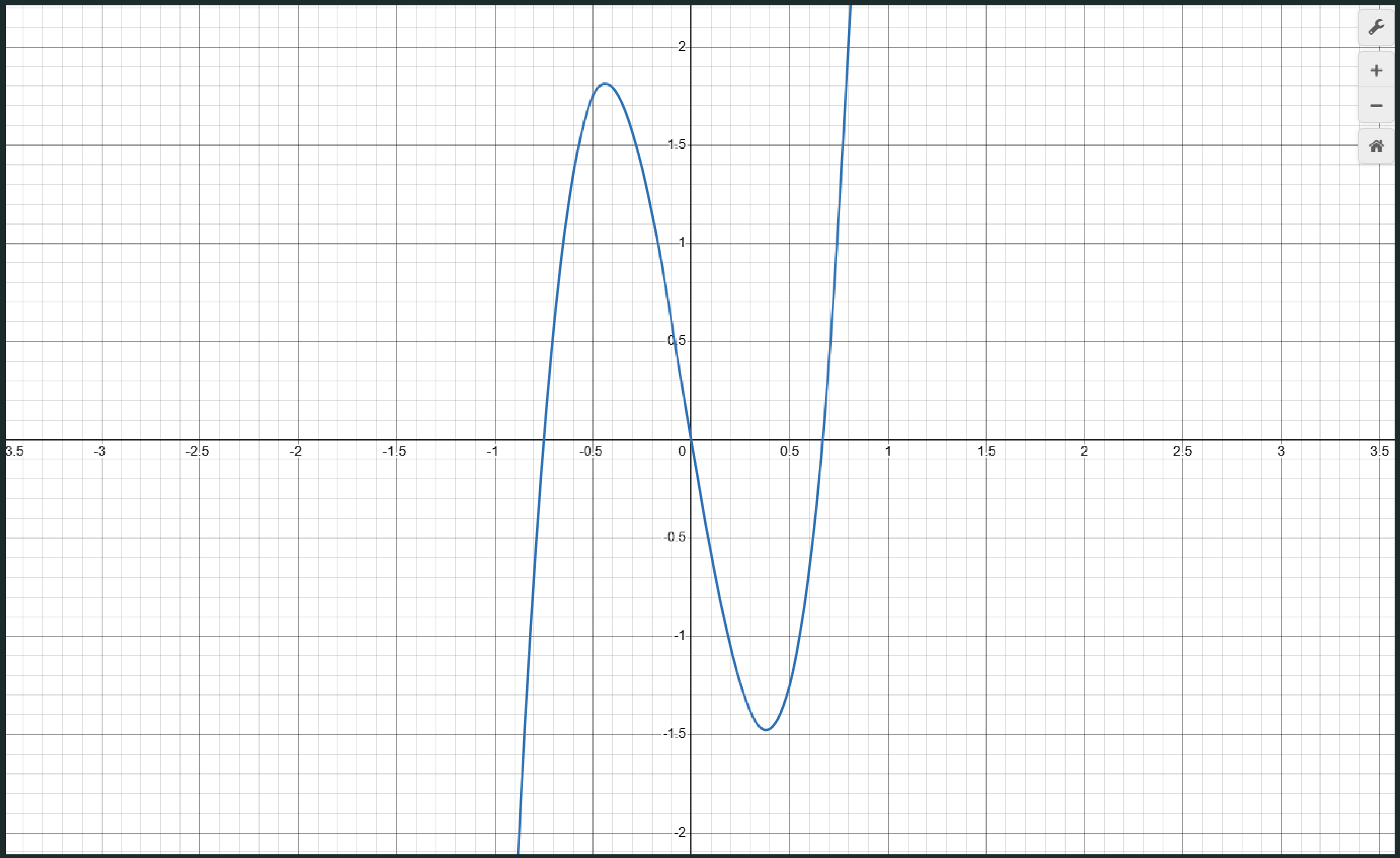
Which of the following represents y as a function of x?

$$x^2 + y^2 = 4$$



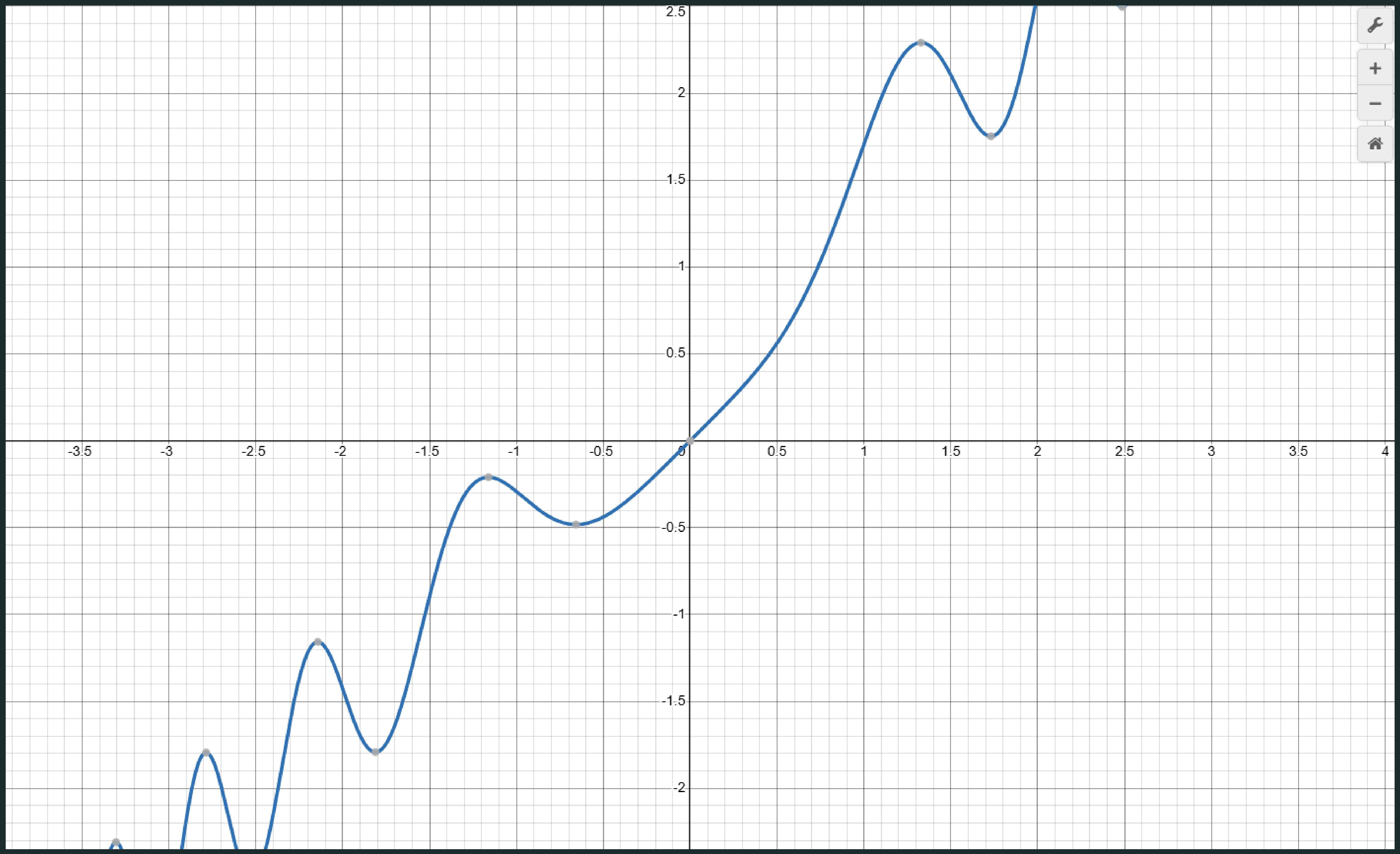
Which of the following represents y as a function of x?

$$y = -6x + x^2 - 12x^3$$



Which of the following represents y as a function of x?

$$y = \sin^2 x^2 + x$$



The notation of a function

If y is a function of x, we represent it by writing

$$y = f(x)$$

where f is the name of the function, x is the independent variable, and $f(x)$ is the dependent variable.

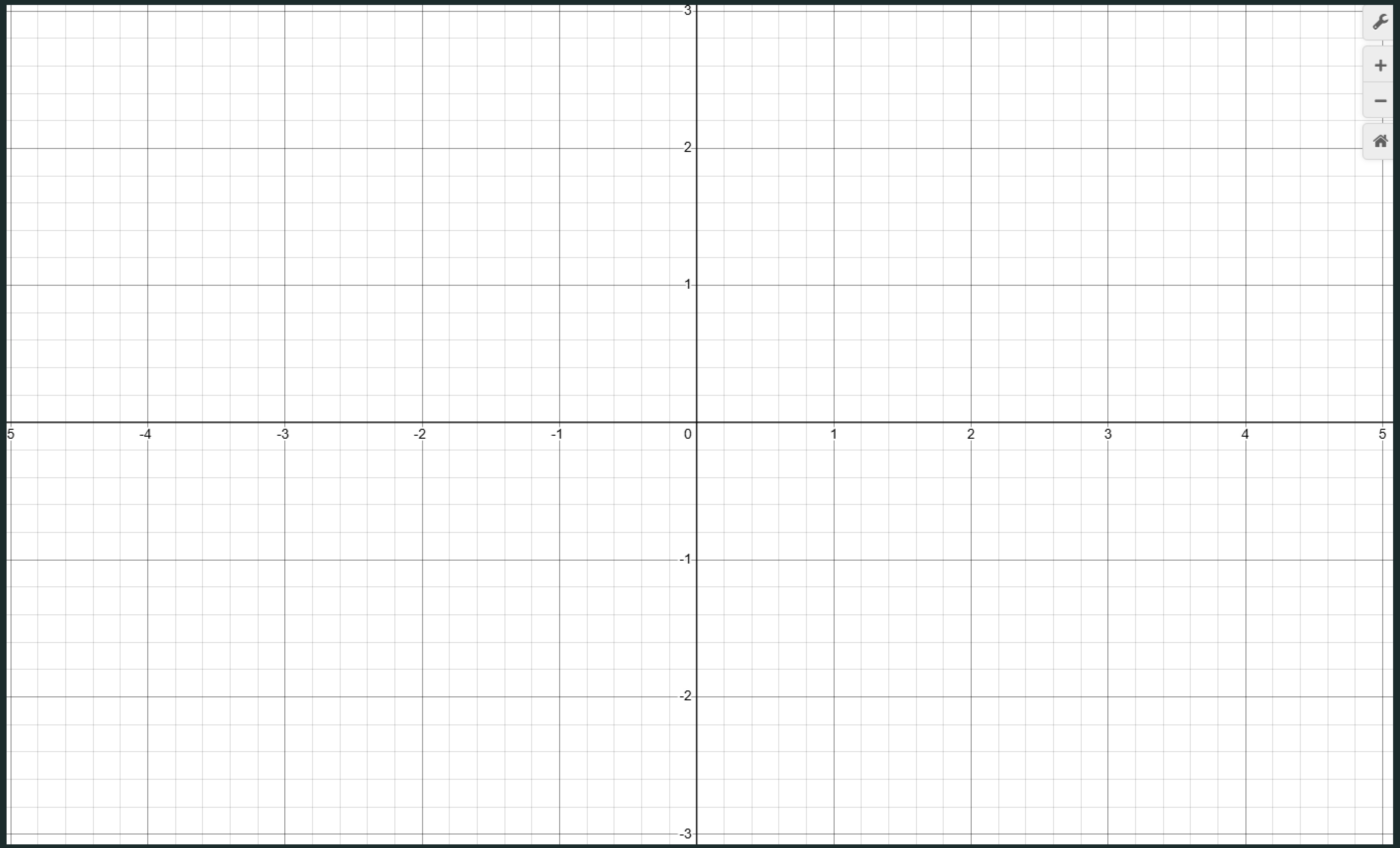
For example,

$$f(x) = x^2 - 1 \qquad \text{is a function of } x,$$

which means that each value of x produces one and only one value of $f(x)$

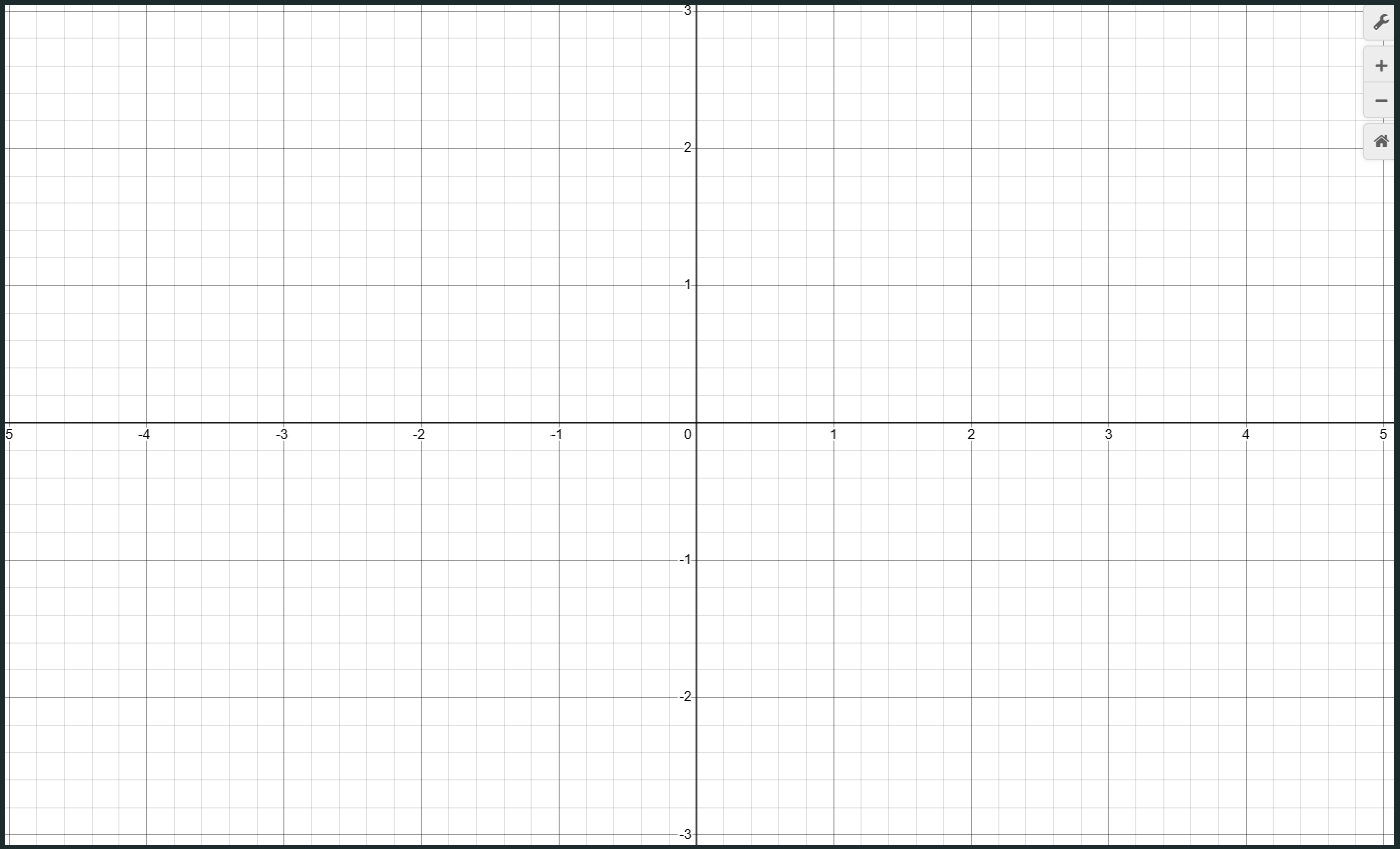
Example

$$f(x) = x^2 - 1$$



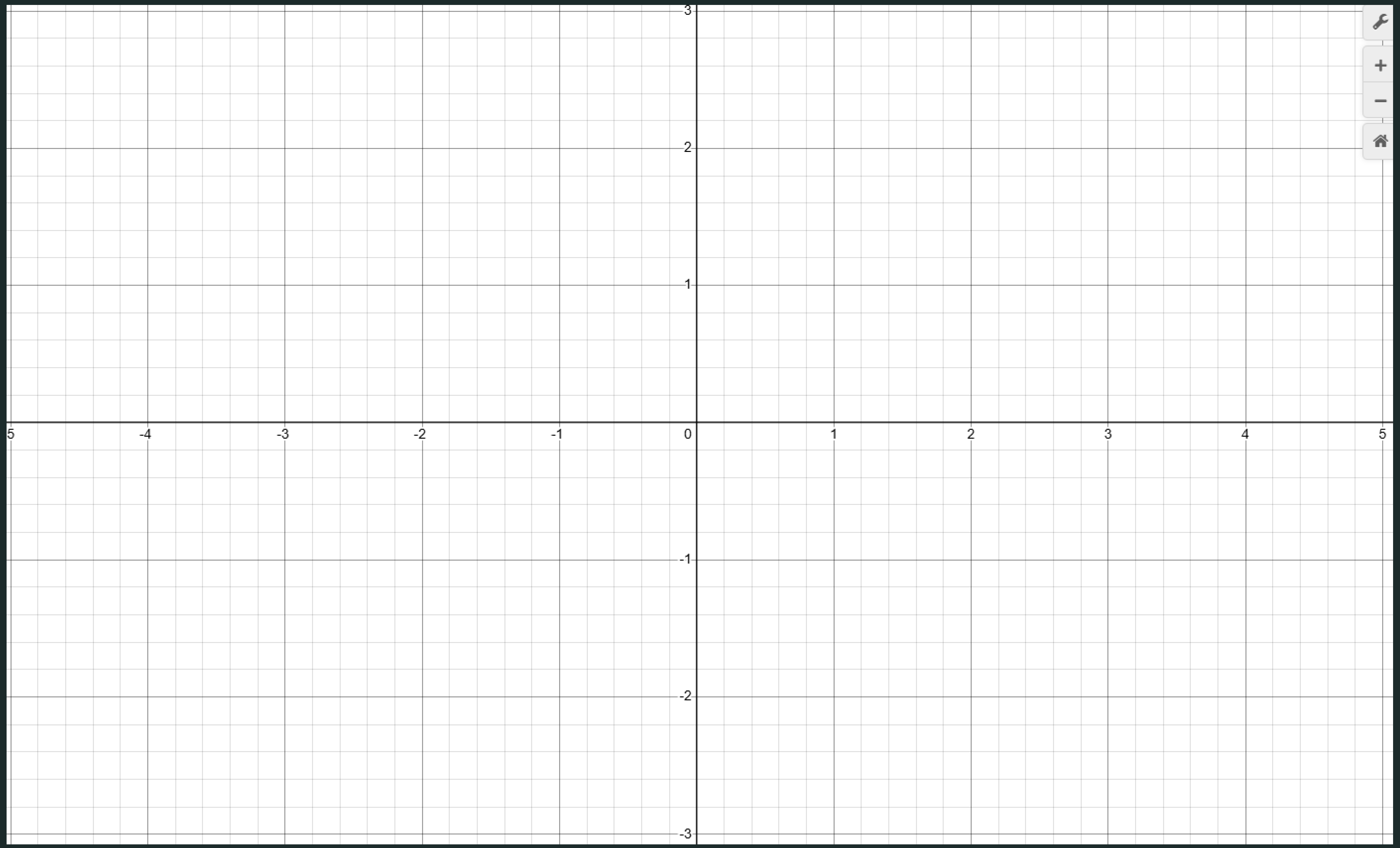
Example

$$f(t) = (-t^2)^2 - 1$$



Example

$$f(a) = a^2 - 1$$



Domain and Range

Domain

The domain of a function is the set of all possible input values (x) for which the function is defined.

For example, you cannot divide by zero or take the square root of a negative number in real numbers

Range

The range of a function is the set of all possible output values (y or f(x)) that result from substituting values of x in the domain.

Example

Determine the domain and range of each of the following functions.

$$f(x)=\sqrt{4-x}$$

$$g(x)=\sqrt{x}-3$$

$$h(x)=\sqrt{x}+\sqrt{x-4}$$

$$k(x)=\frac{1}{\sqrt{1-x^2}}$$

$f(x)=\sqrt{4-x}$

Find Domain

Because there is a square root, the expression inside the root must be nonnegative (≥ 0)

$$4-x \geq 0$$

$$x \leq 4$$

Domain : $(-\infty, 4]$

Find Range

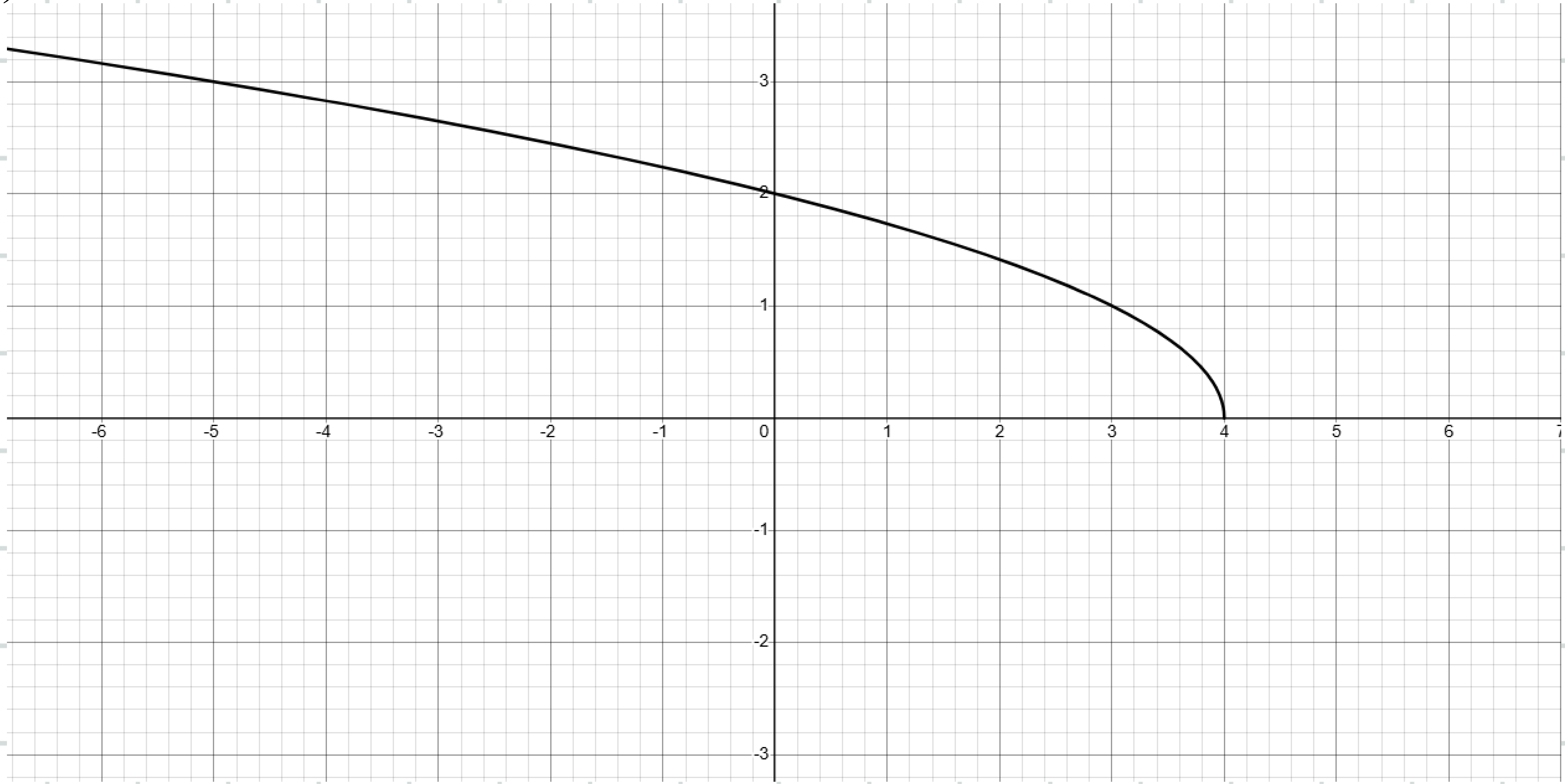
The square root always gives nonnegative results (≥ 0)

When $x = 4$ → $f(4)=0$

As $x \longrightarrow -\infty$ → $f(-\infty) \rightarrow \infty$

Range:$[0,\infty)$

$$f(x)=\sqrt{4-x}$$



$g(x) = \sqrt{x} - 3$

Find Domain

Because there is a square root,

the expression inside the root

must be nonnegative (≥ 0)

$x \geq 0$

Domain : $[0, \infty)$

Find Range

The square root always gives

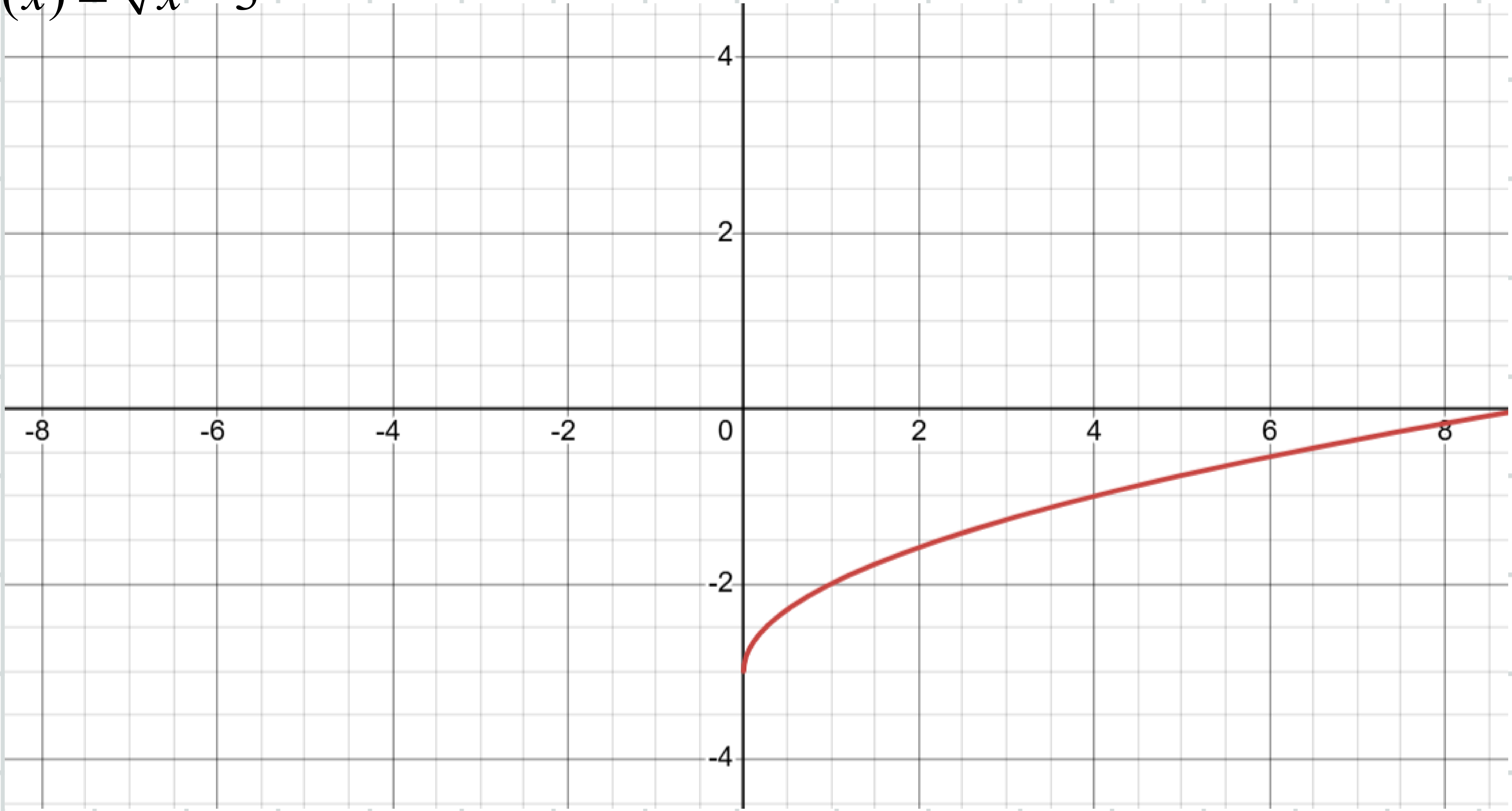
nonnegative results (≥ 0)

When $x = 0$ $\rightarrow$ $g(0) = -3$

As $x \rightarrow \infty$ $\rightarrow$ $g(\infty) \rightarrow \infty$

Range: $[-3, \infty)$

$$g(x) = \sqrt{x} - 3$$



$$h(x)=\sqrt{x}+\sqrt{x-4}$$

Find Domain

Both square roots must be defined (the expressions inside must be nonnegative)

$x \geq 0 \qquad x-4 \geq 0$

$x \geq 4$

Domain : $[4,\infty)$

Find Range

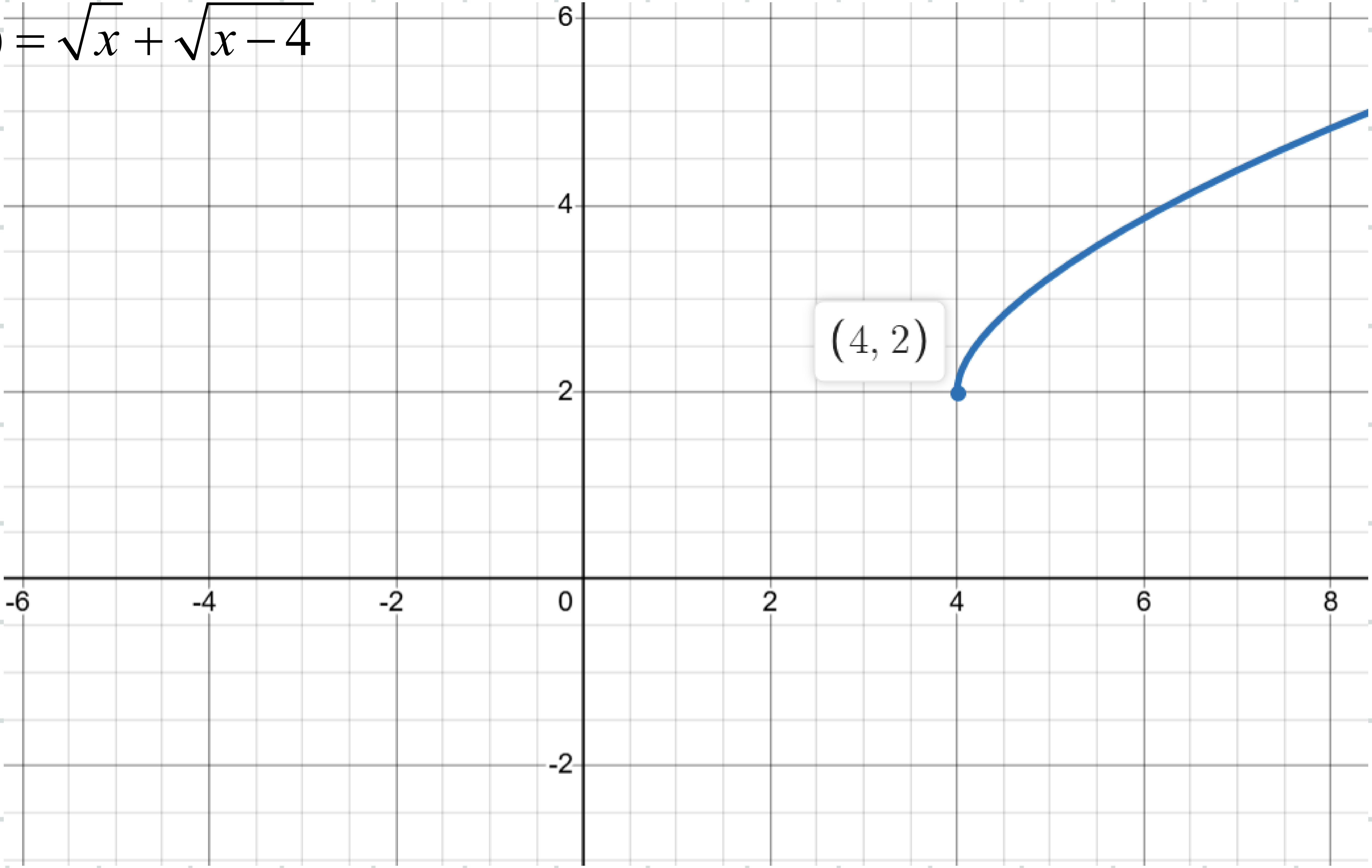
The square root always gives nonnegative results (≥ 0)

When $x=4$ → $h(4)=2$

As $x \to \infty$ → $h(\infty) \to \infty$

Range: $[2,\infty)$

$$h(x)=\sqrt{x}+\sqrt{x-4}$$



$$k(x)=\frac{1}{\sqrt{1-x^2}}$$

Find Domain

The expression inside the square root must be positive (because it's in the denominator — it cannot be zero).

$$1-x^2>0$$

$$x^2<1$$

$$-1<x<1$$

Domain : $(-1,1)$

Find Range

Let's analyze the possible values of $k(x)$

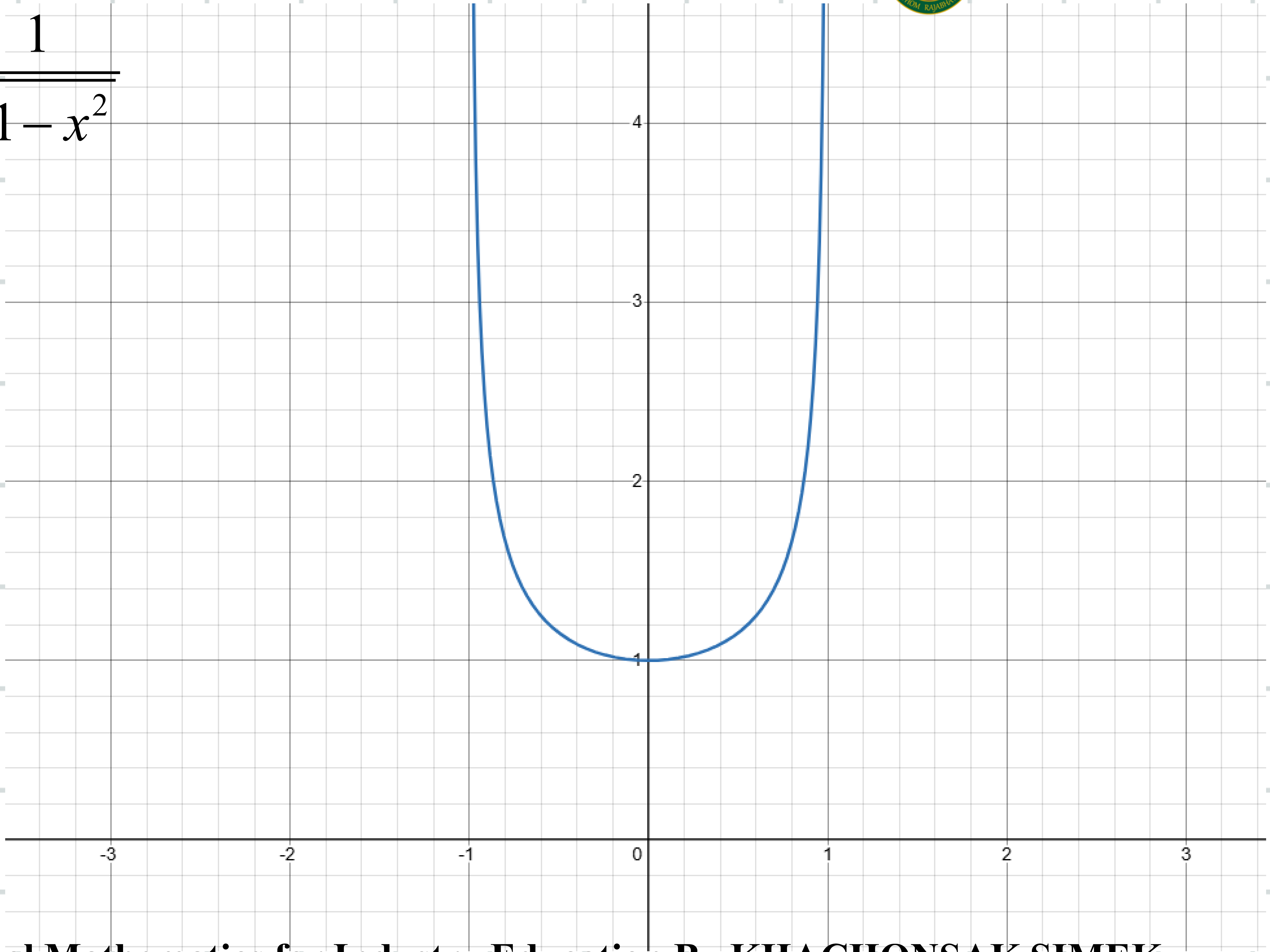
When $x = 0$ → $k(0)=1$

As $x \to 1$ → $k(\to 1)\to\infty$

As $x \to -1$ → $k(\to -1)\to\infty$

Range: $[0,\infty)$

$$k(x)=\frac{1}{\sqrt{1-x^2}}$$



Polynomial functions

$$\boxed{f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0}$$

where n is a non-negative integer, and $a_0, a_1, a_2, \ldots, a_n$ are constants.

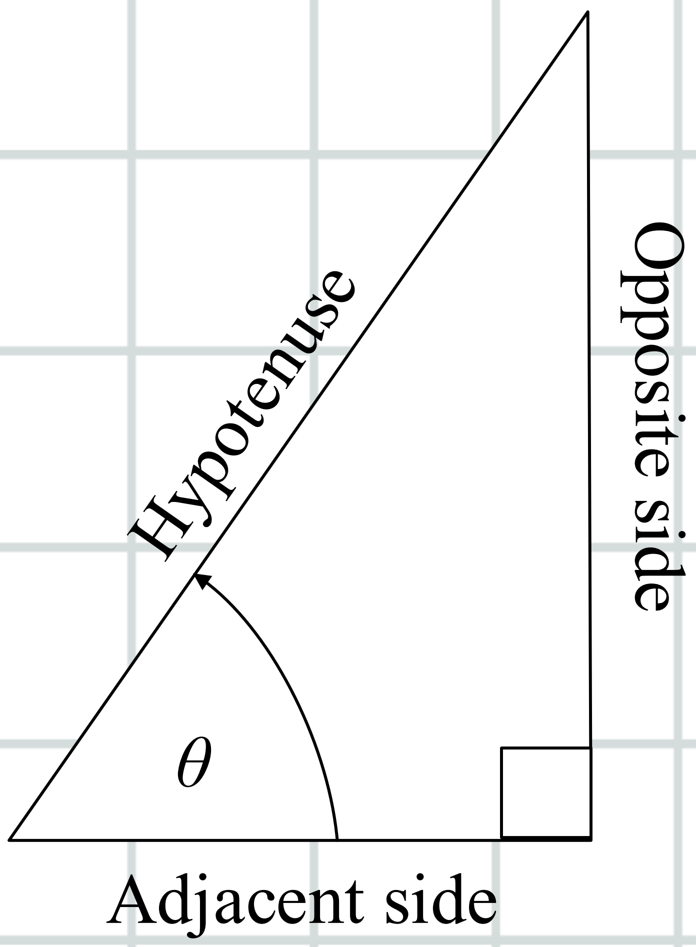
If $a_n \neq 0$, then f is called a polynomial function of degree n.

Example

$f(x) = 2x + 5$	Degree 1	
$g(x) = 3x^2 + 4x - 2$	Degree 2	
$h(x) = x^5 - x^3 + 4$	Degree 5	$h(x) = x^5 + 0x^4 - x^3 + 0x^2 + 0x + 4$
$k(x) = 7$	Degree 0	
$l(x) = 0x^4 + 8x^3 + x^2 + x + 77$	Degree 3	

Trigonometric functions

A function of an angle that describes the relationship between the angle and the ratios of the sides of a right triangle.



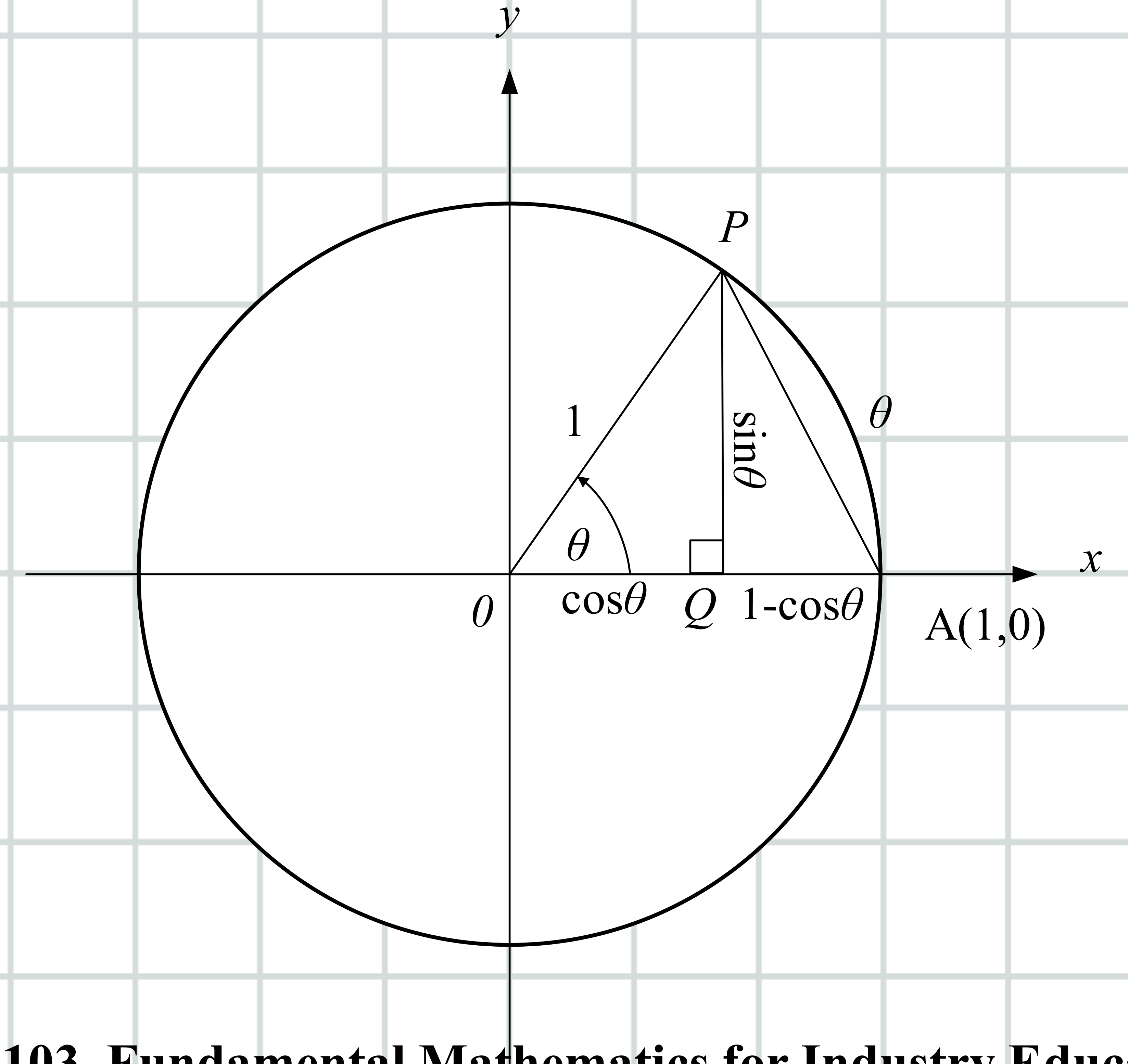
It can also be defined as a function that expresses the relationship between an angle and the coordinates of a point on the unit circle.

Function	Definition in a Right Triangle
$\sin\theta$	= Opposite side / Hypotenuse
$\cos\theta$	= Adjacent side / Hypotenuse
$\tan\theta$	= Opposite side / Adjacent side

The other three functions are defined as reciprocals of these:

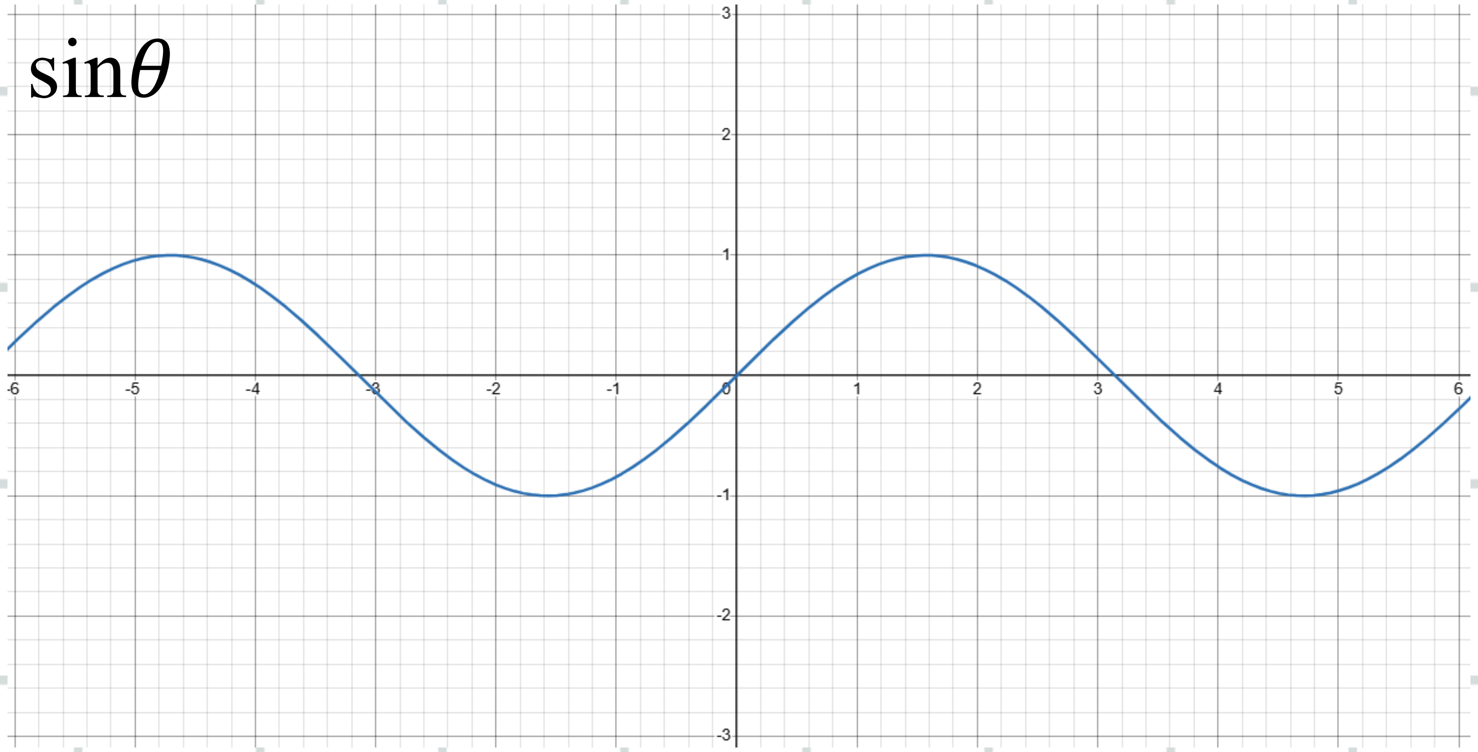
$$\csc\theta = \frac{1}{\sin\theta},\ \sec\theta = \frac{1}{\cos\theta},\ \cot\theta = \frac{1}{\tan\theta}$$

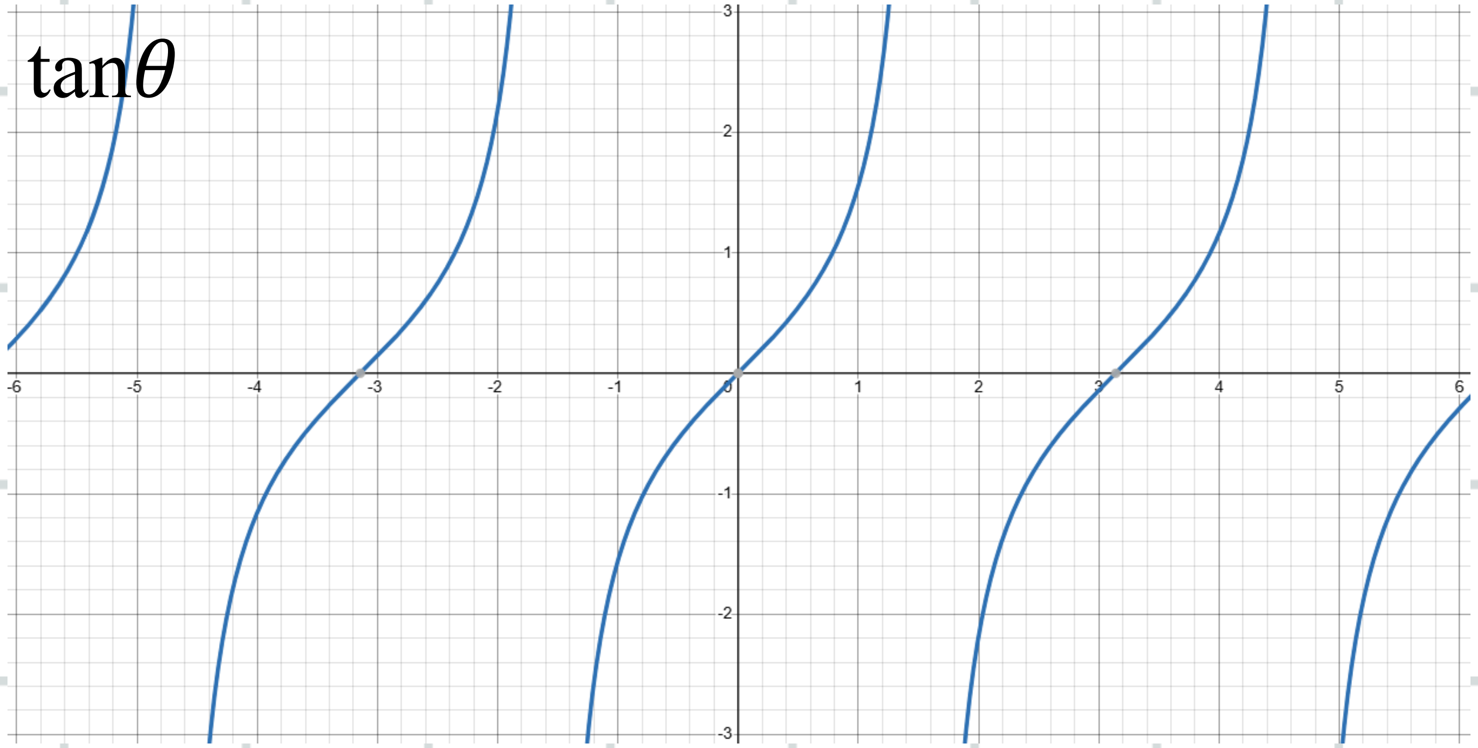
Trigonometric functions

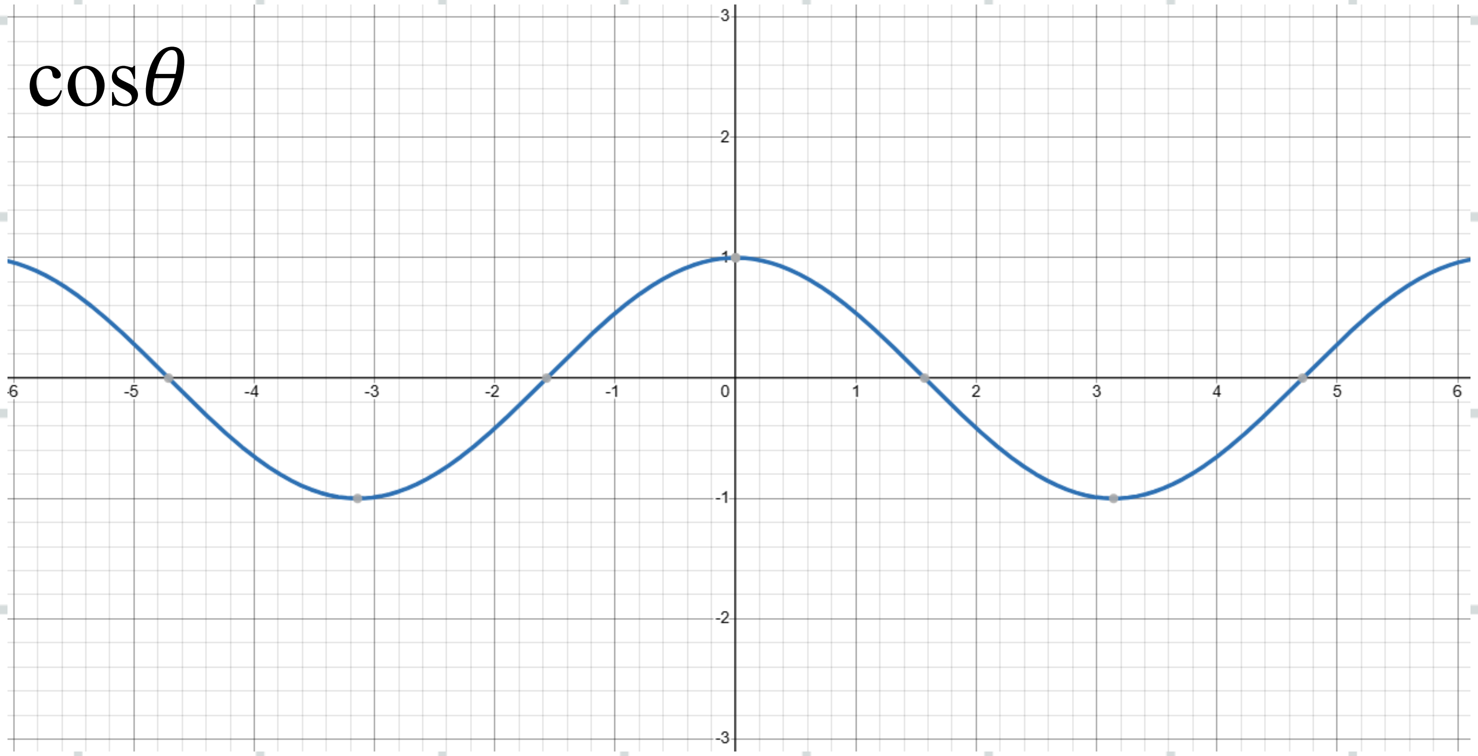


On the Unit Circle. If a point $P(x,y)$ lies on a circle of radius 1 centered at the origin (0,0) and the line from the origin to P makes an angle θ with the x-axis

then $\sin\theta = y,\ \cos\theta = x\ ,\tan\theta = \dfrac{y}{x}$


$\sin\theta$


$\tan\theta$


$\cos\theta$

Exponential Functions

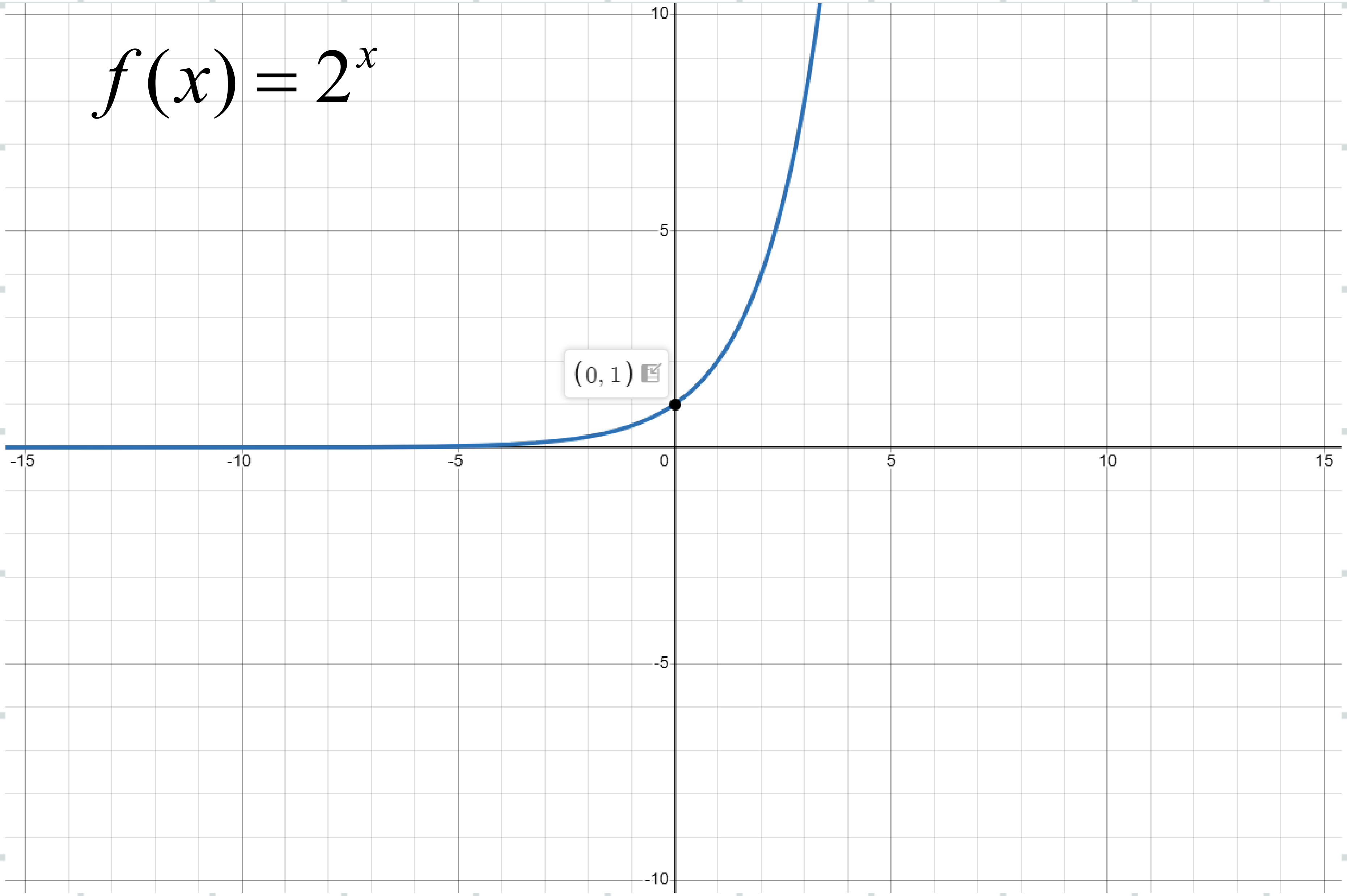
A function in which the variable appears in the exponent of a constant base.

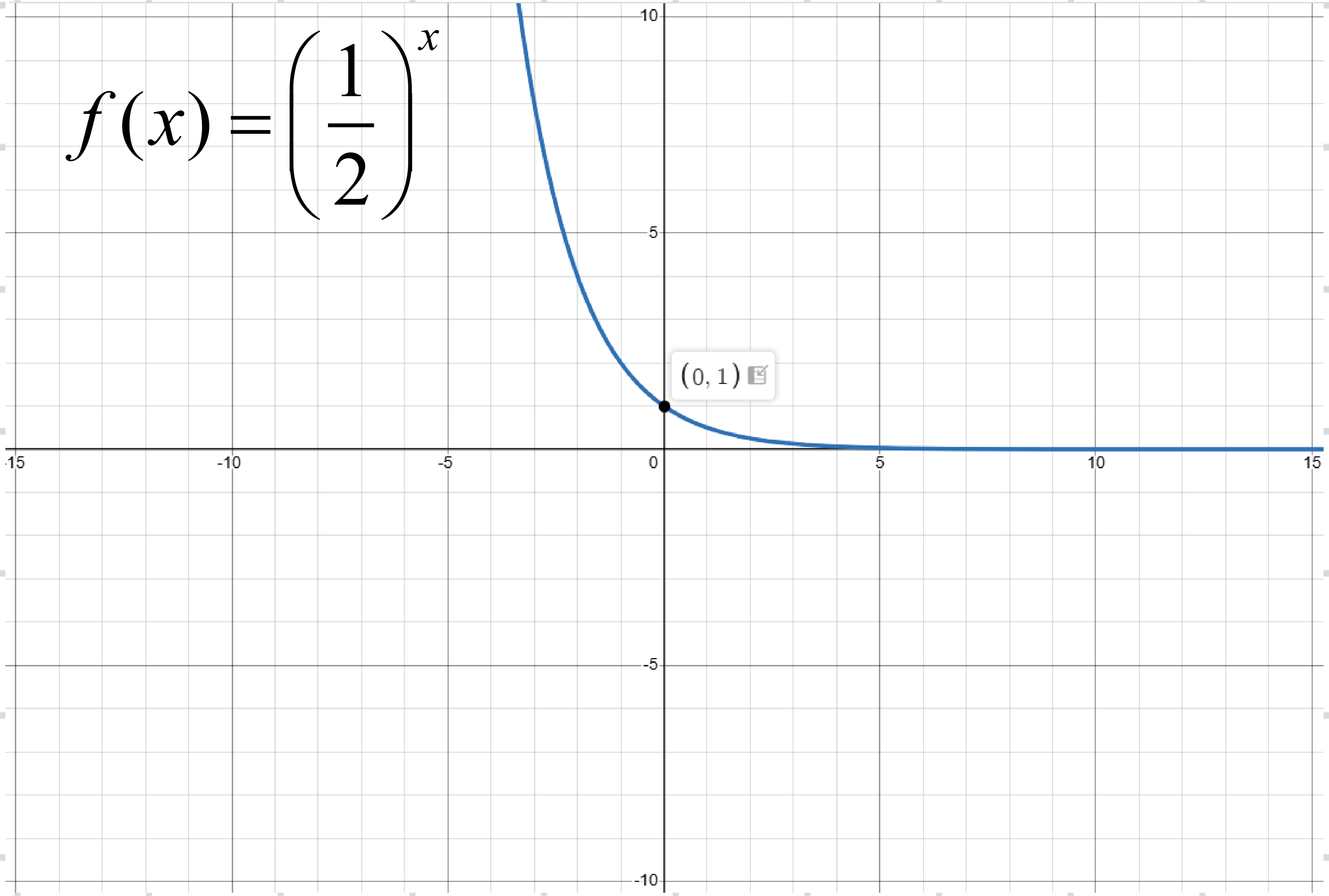
It is generally written as:

$$f(x) = a^x$$

Properties

1. Domain: all real numbers $(-\infty, \infty)$
2. Range: all positive numbers $(0, \infty)$
3. If $a > 1$, the function is increasing.
4. If $0 < a < 1$, the function is decreasing.
5. The graph always passes through the point $(0,1)$ since $a^0=1$


$f(x) = 2^x$
(0, 1)


$f(x) = \left(\frac{1}{2}\right)^x$
(0, 1)

Logarithmic Functions

the inverse of an exponential function.

It is generally written as:

$$f(x) = \log_a x$$

Where a is a positive real constant and a $\neq 1$.

Relation to the Exponential Function

If

$$y = \log_a x$$

then it means

$$a^y = x$$

Properties

1. Domain: x > 0
2. Range: (- ∞, ∞)
3. The graph always passes through the point (0,1) because $\log_a 1 = 0$

Summary

- A function is written as $y = f(x)$ read as “f of x”
- This means y is the value obtained by substituting x into the rule of the function.
- Domain: all possible input values of x for which the function is defined.
- Range: all possible output values $y = f(x)$ produced by the function.

Exercise

1. A metal fabrication factory wants to design a rectangular steel plate. The length of the plate is 1.75 times its width (w).Write a function that represents the area (A) of the steel plate in square meters and determine the domain and range of this function.
2. Evaluate the function $f(x) = x^2 + 2x + 1$ When x= -2, 0, 1, k
3. Evaluate the function $g(x) = \sin 3x$ When x= 0, 10, 15, 30