Welcome to

FUNDAMENTAL MATHEMATICS

FOR INDUSTRY EDUCATION

Lesson 2

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Systems of Equations

A group of two or more equations that share common variables and are solved simultaneously to find a single set of variable values that satisfies all equations at the same time.

Classification of Systems of Equations

- **Linear systems of equations**
- **Non-linear systems of equations**
- **Polynomial systems of equations**
- **Systems of equations with multiple variables**
- **Systems of equations in matrix form**

Linear equations

Linear Equation in One Variable

A linear equation in one variable is an equation that can be written in the form

$$Ax + B = 0$$

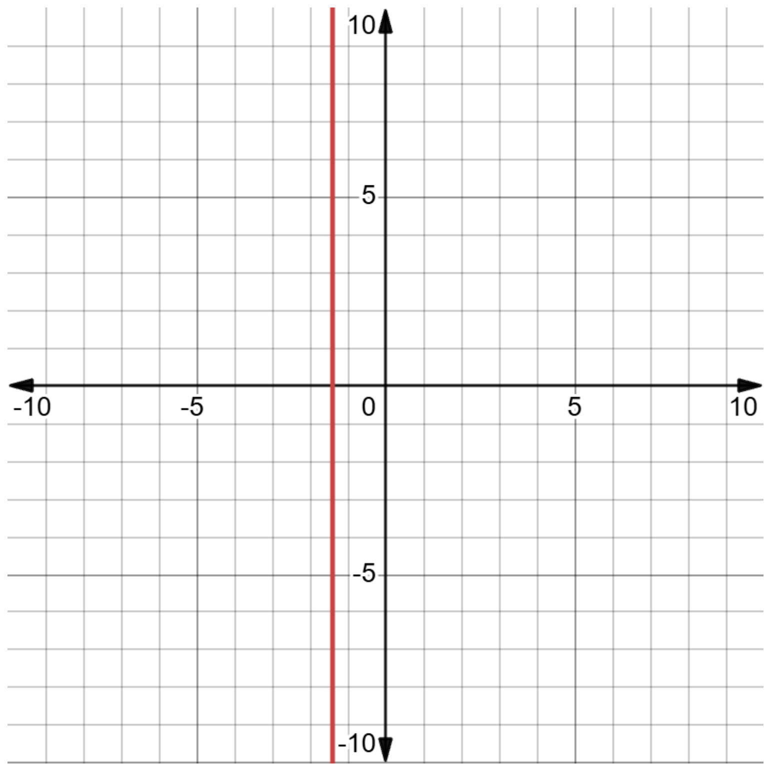
where A and B are constants and $A \neq 0$

Linear equations

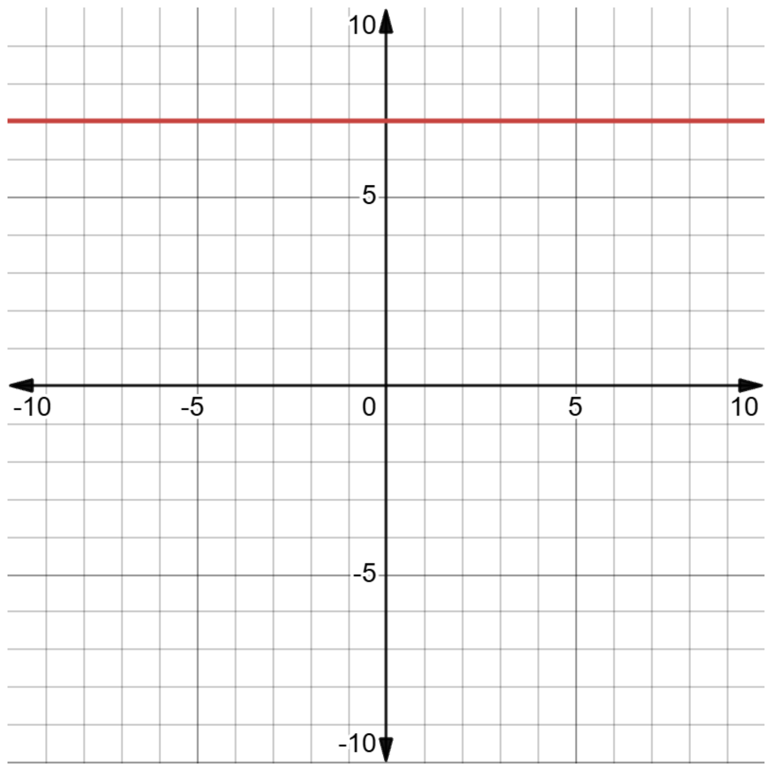
Linear Equation in One Variable

Example

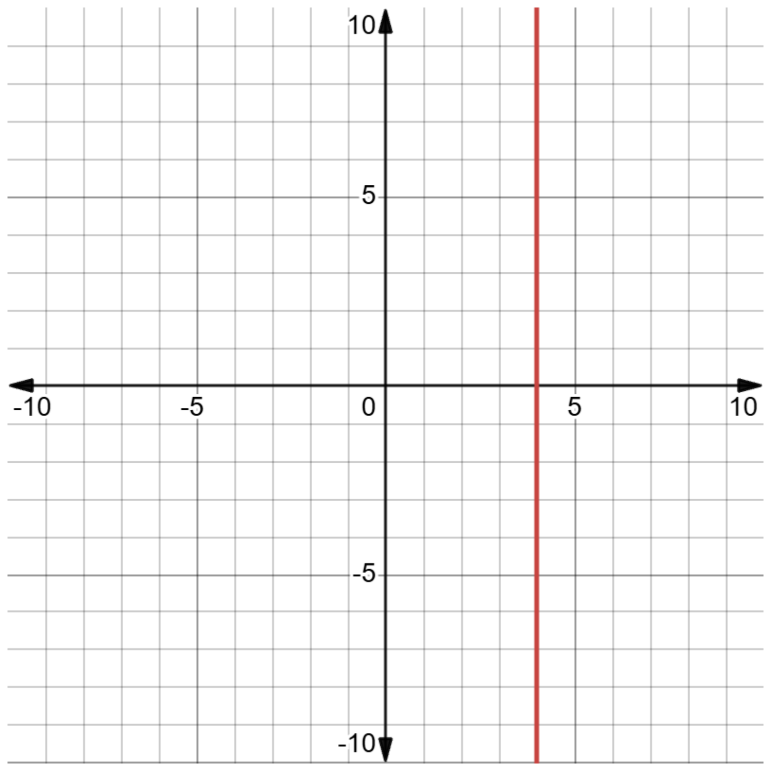
$$5x+7=0$$



$$y=7$$



$$\frac{3}{2}x-6=0$$



Linear equations

Linear Equation in Two Variables

A linear equation in two variables is an equation that can be written in the form

$$Ax + By + C = 0$$

Where *A, B,* and *C* are constants, and *A* and *B* are not equal to zero.

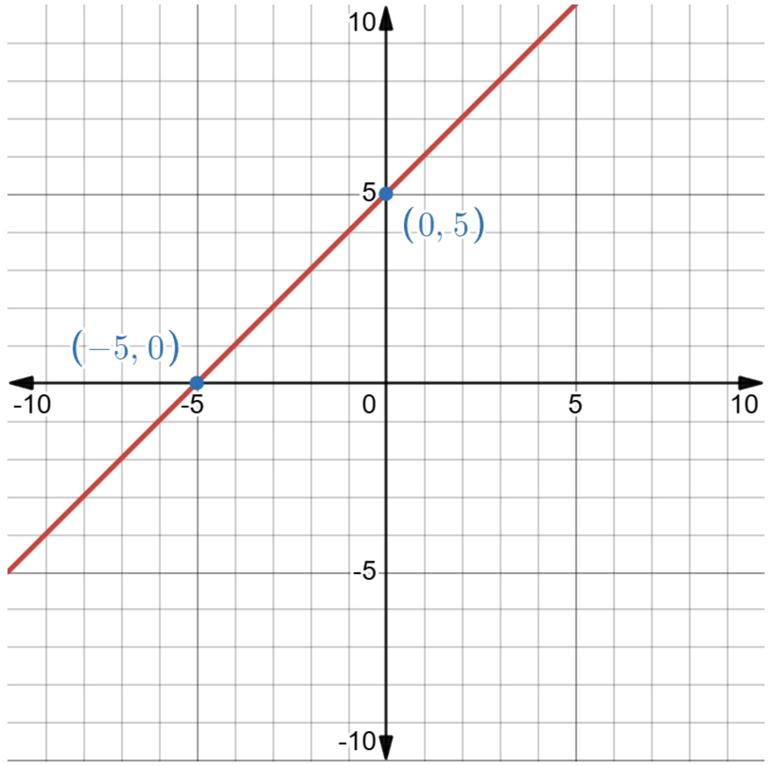
Linear equations

Linear Equation in Two Variables

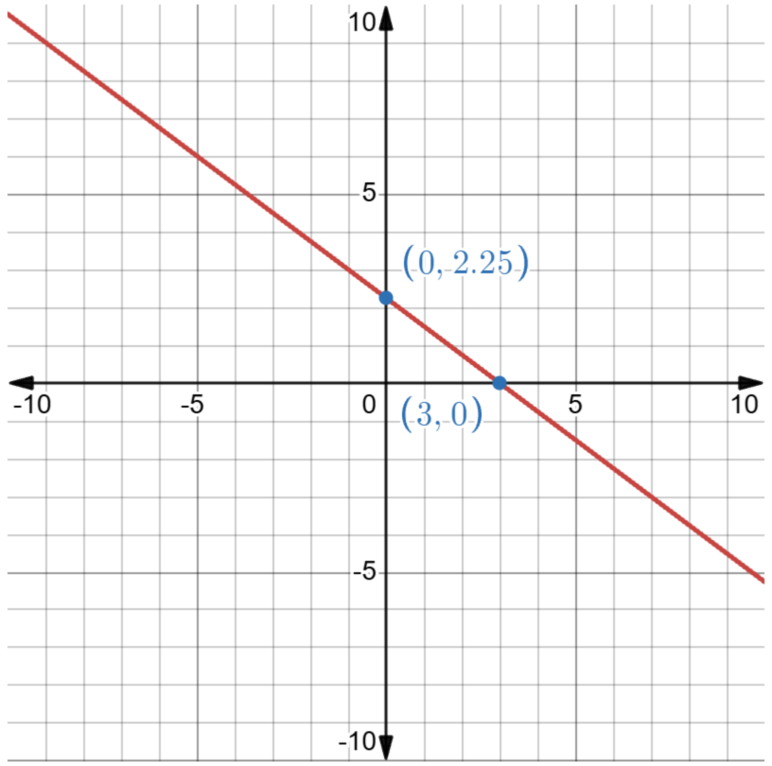
Where *A*, *B*, and *C* are constants, and *A* and *B* are not equal to zero.

Example

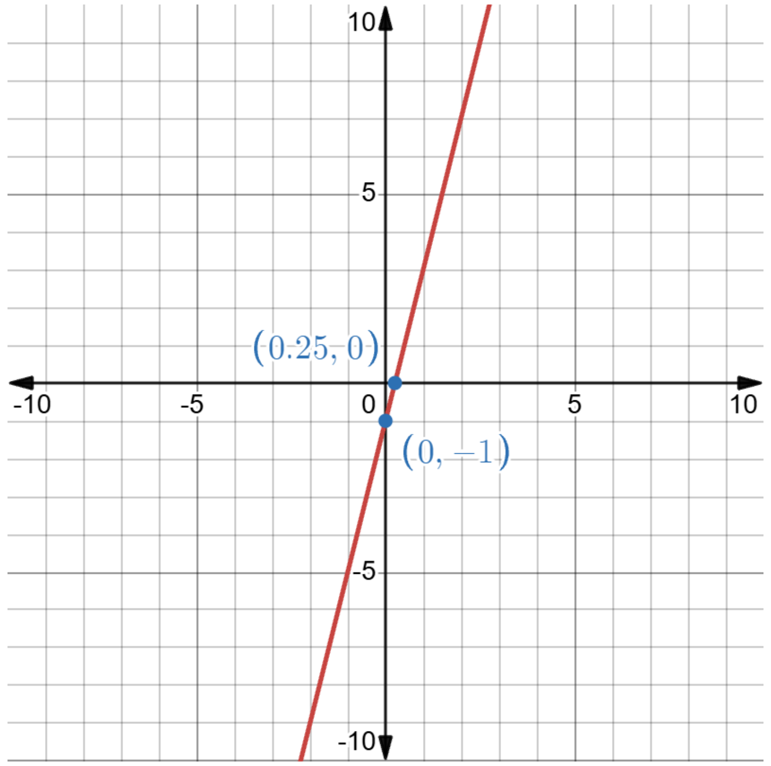
$$x - y + 5 = 0$$



$$3x + 4y - 9 = 0$$



$$y = 4x - 1$$



Linear equations

Linear Equation in Two Variables

Standard Form

$$y = mx + c$$

Where m is the slope of the line, and c is the y-intercept, the point where the line crosses the y-axis.

*Important Characteristics of Linear Equations in Two Variables

1. Both variables are of degree 1.
2. The variables do not appear in fractional form.
3. A and B are not zero at the same time.
4. When plotted, the graph is a straight line.

Linear equations

Linear Equation in Two Variables

Techniques for Graphing Linear Equations in Two Variables

Step 1: Find the x-intercept by letting $y = 0$ and solving for x.

Step 2: Find the y-intercept by letting $x = 0$and solving for y.

Step 3: Draw a straight line passing through the x-intercept and the y-intercept.

Example

Draw the graph of the linear equation $3x - 2y = 6$

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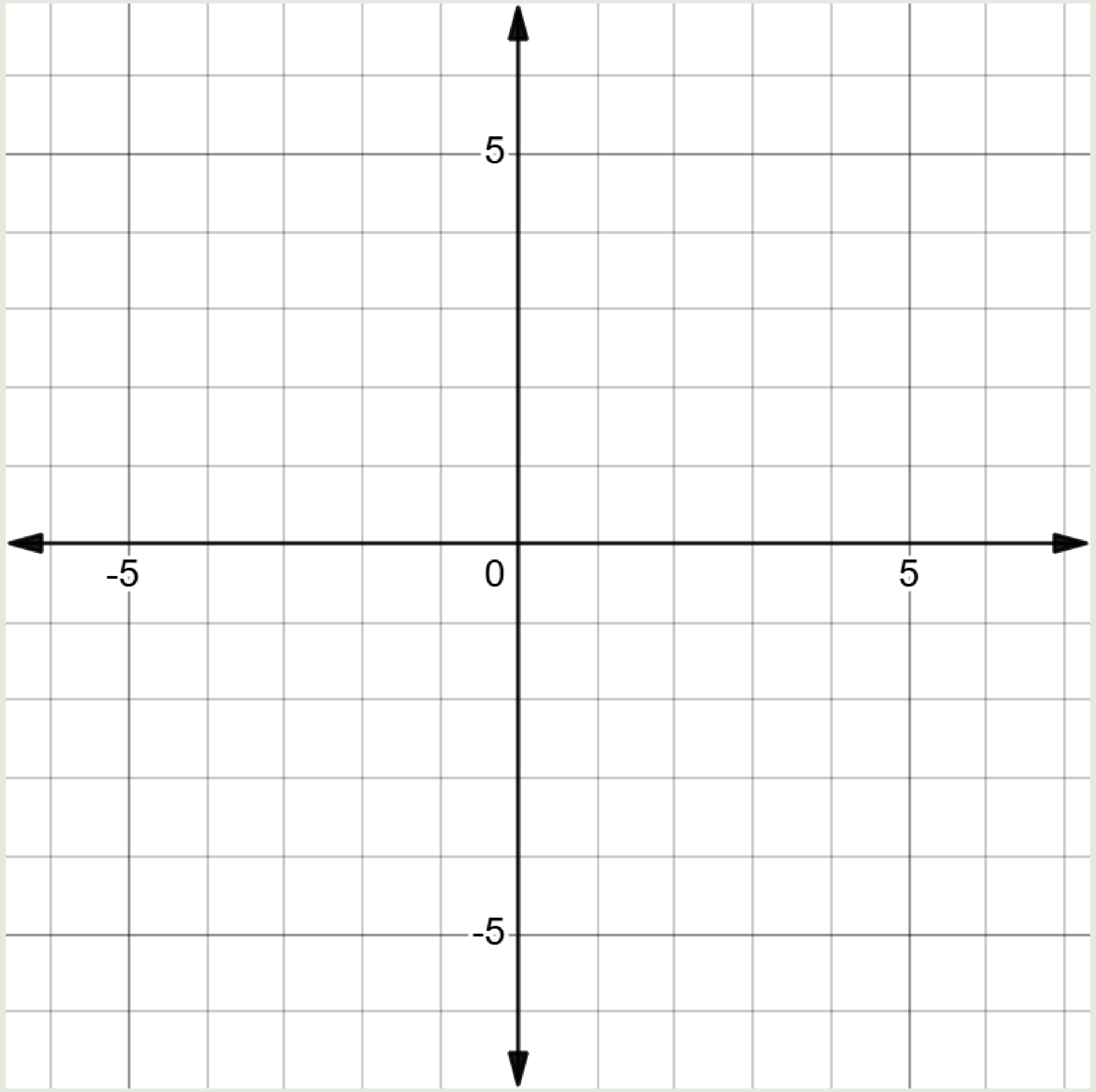


Linear equations

Linear Equation in Two Variables

Slope of a Line

Let L be a straight line passing through the points (x_1,y_1) and (x_2,y_2)

where $x_1 \neq x_2$

The slope m of the line L is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{\Delta y}{\Delta x}, x_1 \neq x_2$$

Example

Find the slope of the line passing through the points(2,3) and (6,11)

Example

Find the slope of the line $2x + 4y = 8$

Polynomial equations

Quadratic Equation

An equation that can be written in the form

$$Ax^2 + By^2 + Cxy + Dx + Ey + F = 0$$

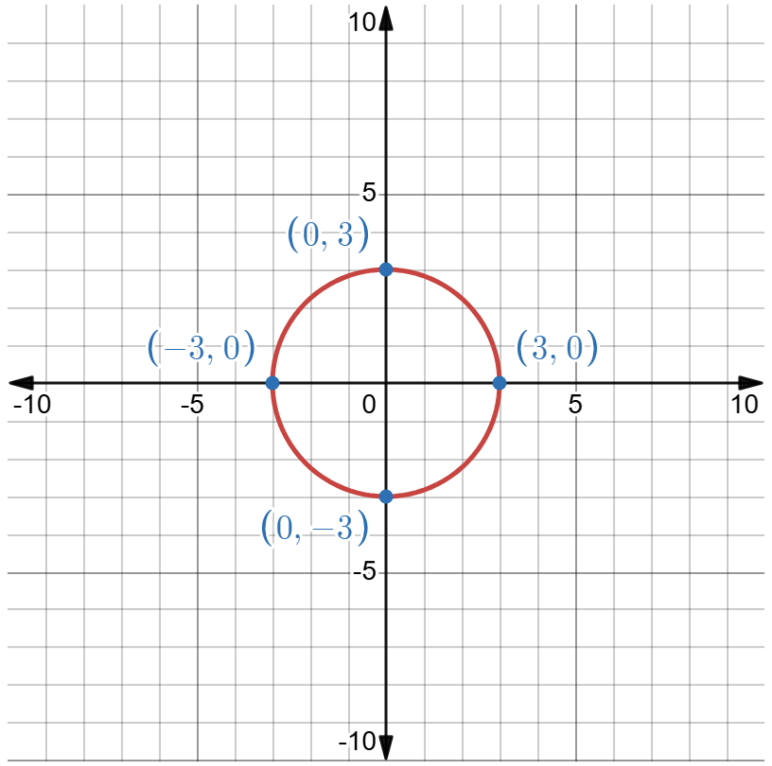
where *A*, *B*, *C*, *D*, *E*, and *F* are constants, and *A*, *B*, and *C* are not all equal to zero at the same time

Polynomial equations

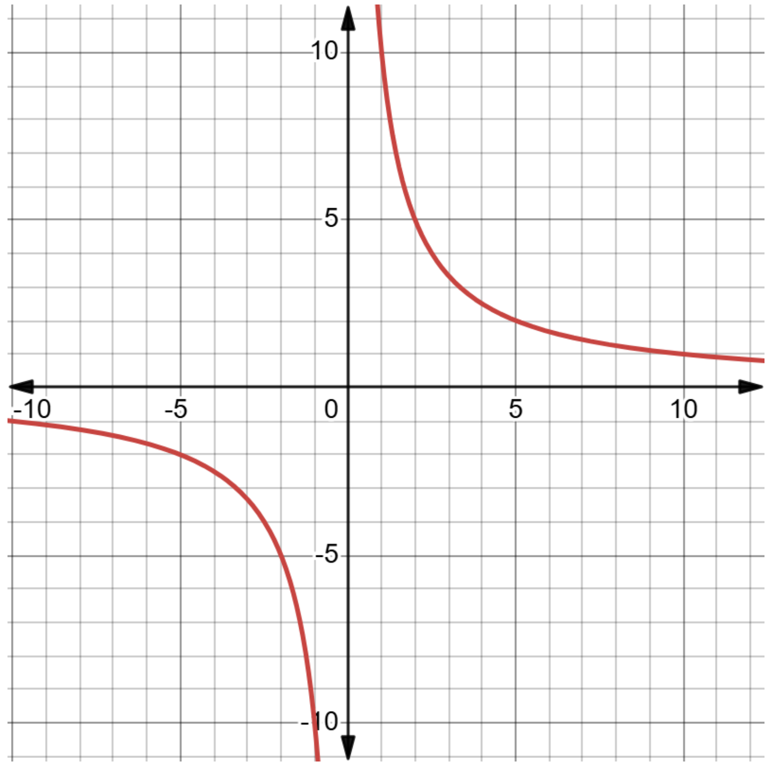
Quadratic Equation

Example

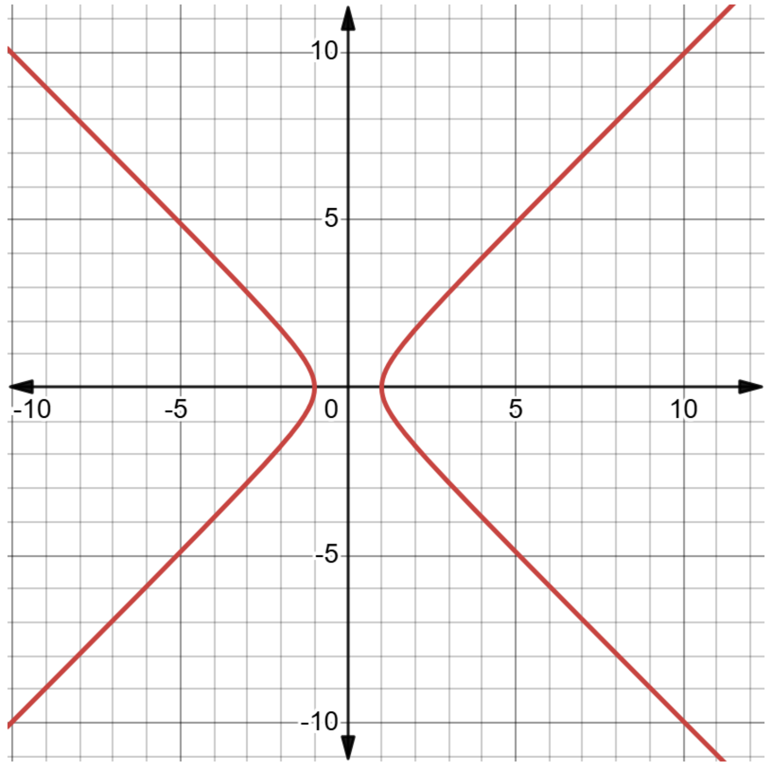
$$x^2 + y^2 = 9$$



$$xy = 10$$



$$x^2 - y^2 - 1 = 0$$



Solving Systems of Equations

Finding the solution of the system.

The solution is the **point of intersection of the graphs** of the equations in the system.

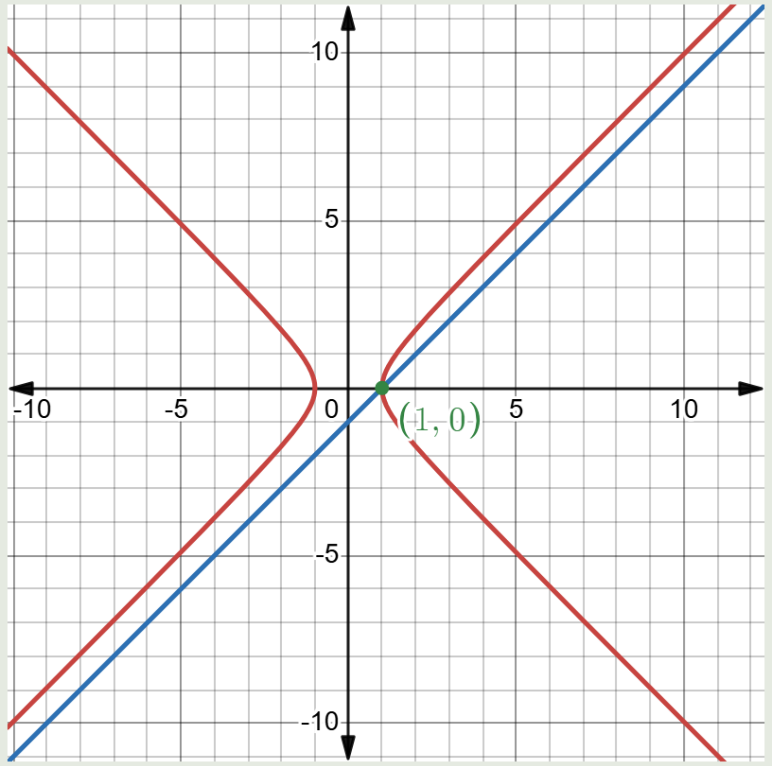
There are **three possible types of solutions** as follows:

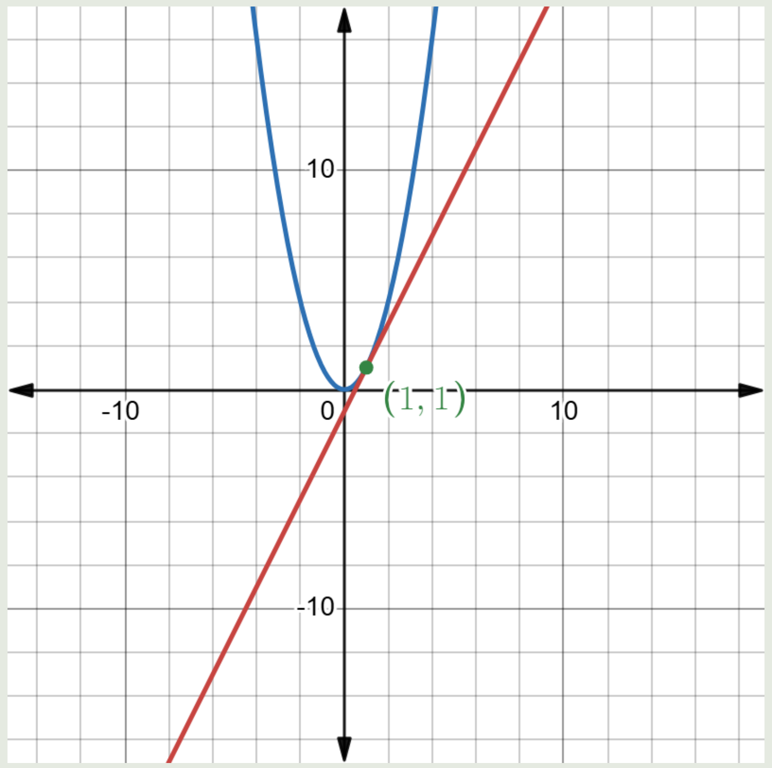
One point of intersection

the system of equations has one solution

$$x^2 - y^2 = 1$$
$$y = x - 1$$

$$x^2 - y = 0$$
$$2x - y = 1$$





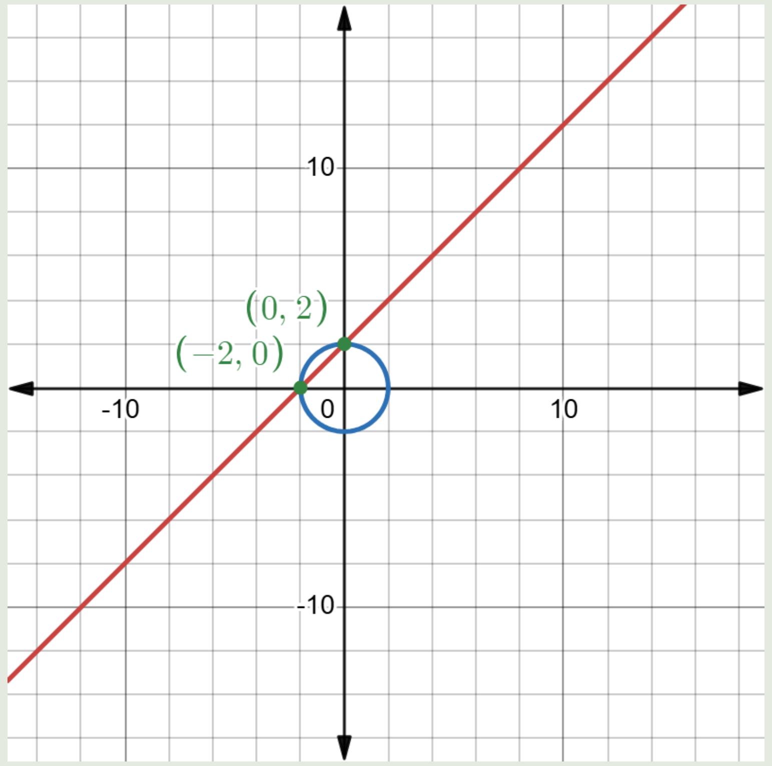
Solving Systems of Equations

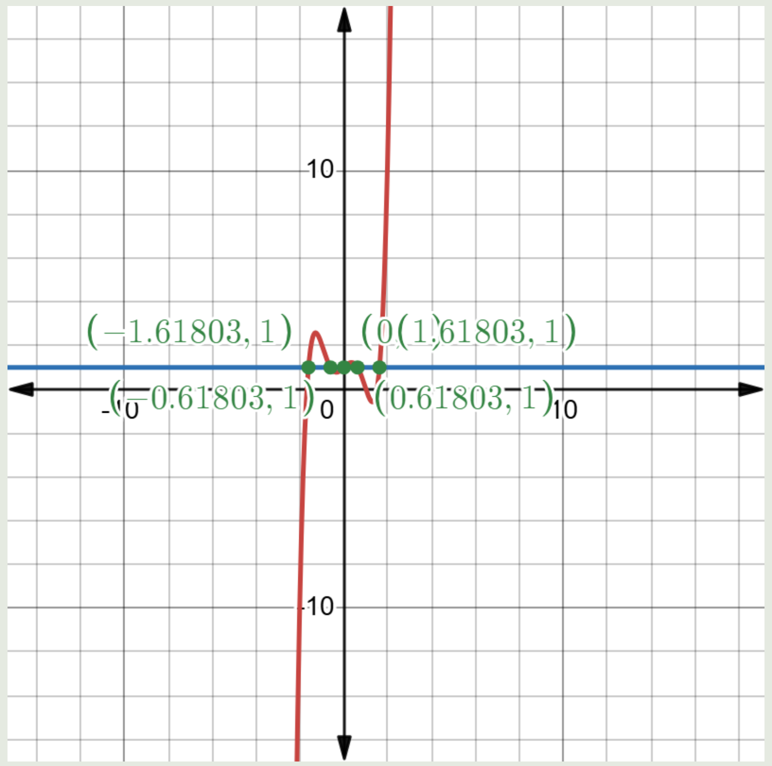
More than one point of intersection

the system of equations has more than one solution

$$x^2 + y^2 = 4$$
$$y = x + 2$$

$$y = 1$$
$$y = (x^5 - 3x^3) + x + 1$$



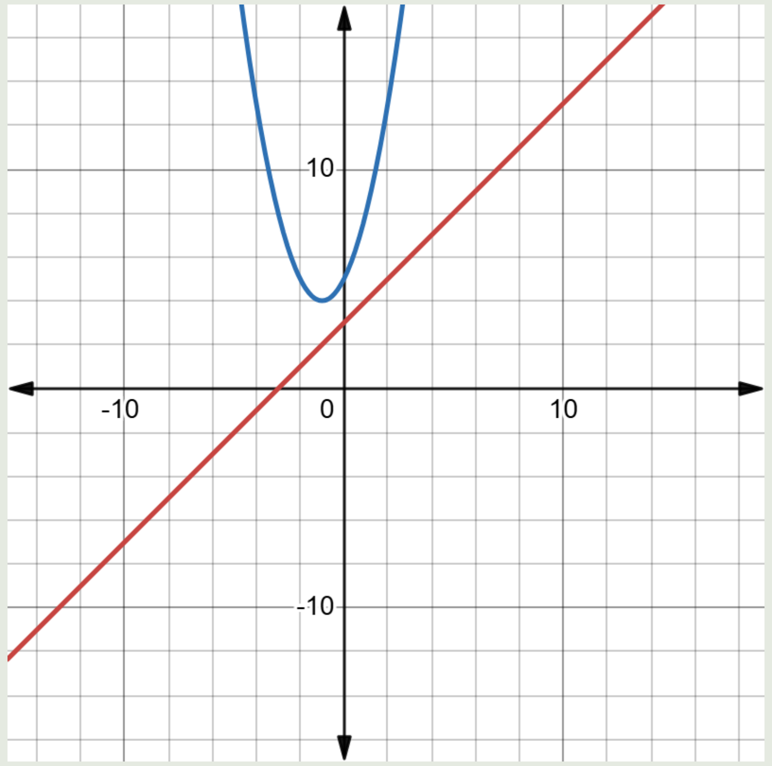


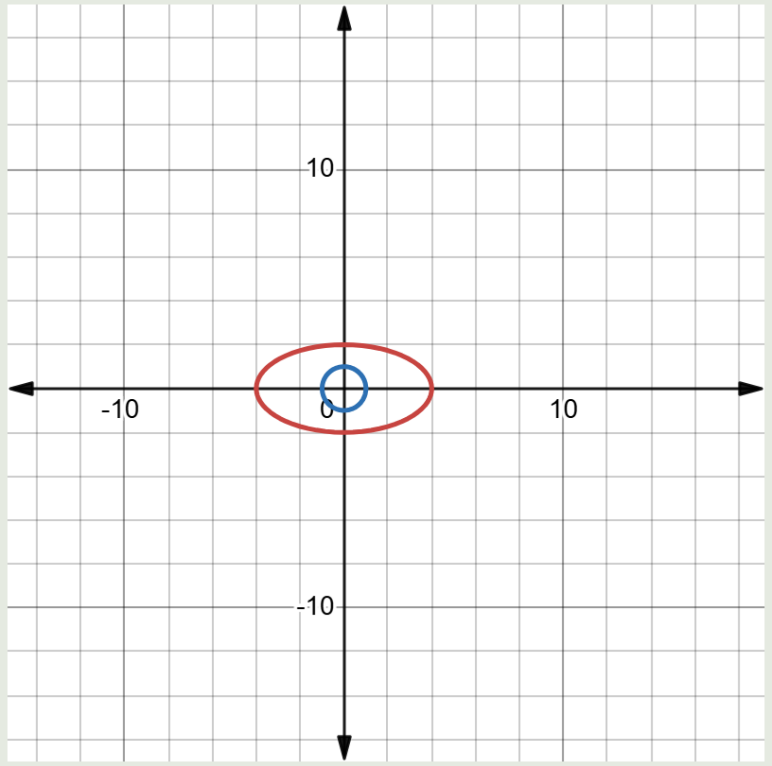
No point of intersection

the system of equations has no solution

$$y - x = 3$$
$$y = x^2 + 2x + 5$$

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$
$$x^2 + y^2 = 1$$

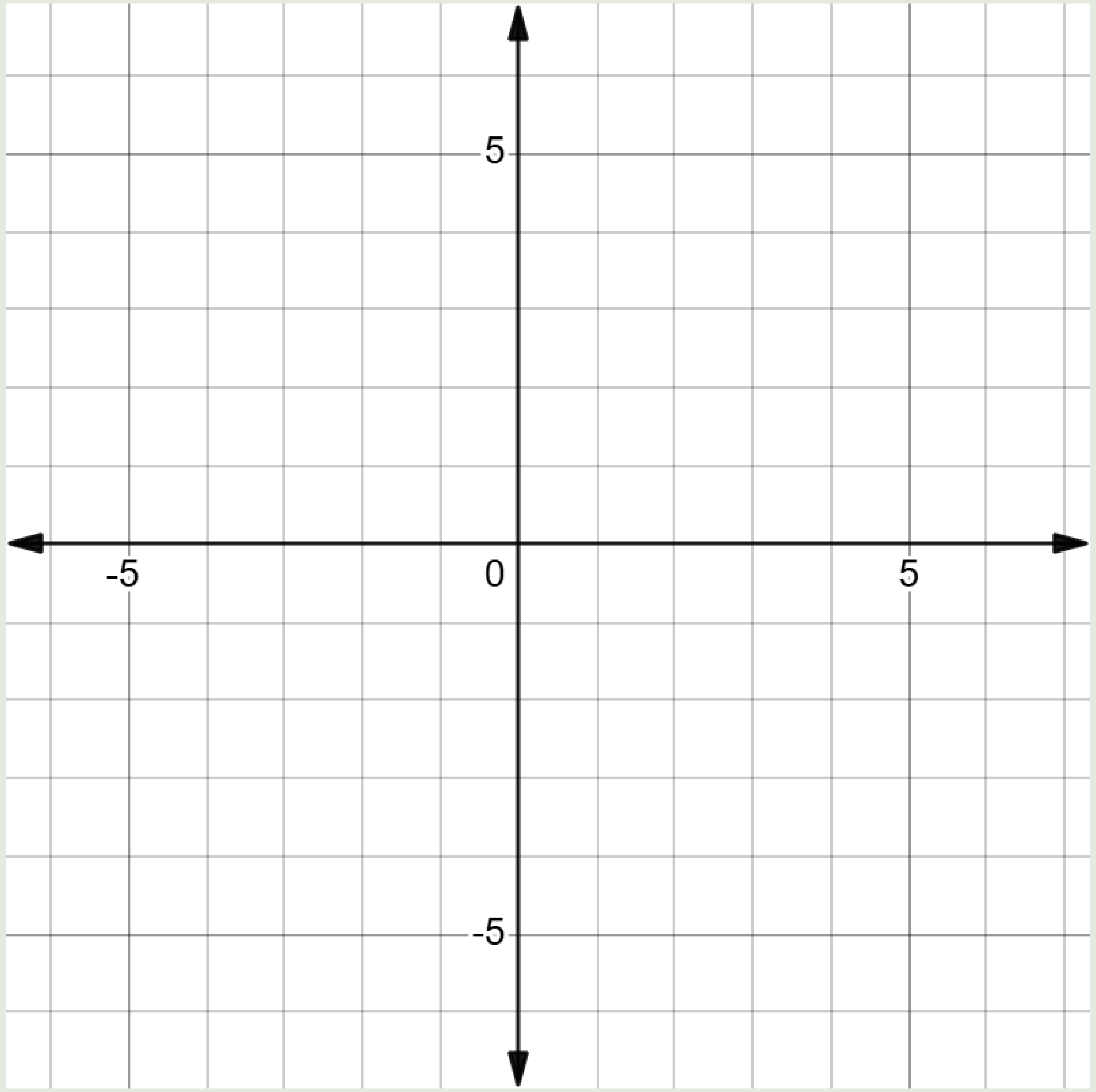




Example

Find the point of intersection of the graphs of the equations

$$y = 2x + 1$$
$$y = -x + 7$$



Common Forms of Systems of Equations in Two Variables

Form 1: A system of equations consisting of **two linear equations**

Form 2: A system of equations consisting of **one linear equation** and **one second-degree (quadratic) equation**

Form 3: A system of equations consisting of **two second-degree (quadratic) equations**

In addition, you may encounter equations of a **degree higher than two**, or **systems of equations with three variables**, which consist of **three equations**.

Method for Solving Systems of Equations: 1.Substitution

Step 1: Choose one equation (If there is a linear equation, choose the linear one.)

Step 2: Rearrange the equation to make one variable the subject:

$y =$ an expression in terms of x,

or $x =$ an expression in terms of y

Example: $2x + 3y = 9$

$$y = \frac{9-2x}{3}$$

or

$$x = \frac{9-3y}{2}$$

Method for Solving : 1.Substitution Method

Step 3: Substitute the expression for x or y into the other equation.

Step 4: Solve the resulting equation to find the value of the first variable.

Step 5: Substitute the value of the first variable back into one of the original equations to find the second variable.

Example

There are 12 bicycles and tricycles in total, and the total number of wheels is 29.Find the number of bicycles and tricycles.

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Method for Solving : 2. Elimination Method

Step 1: Make the coefficients of the variable to be eliminated equal(or make them opposites).

Step 2: If they are equal, subtract the equations. If they are opposites, add the equations.

Step 3: Solve for the first variable.

Step 4: Substitute the value of the first variable into the second equation.

Step 5: If the system has two equations of degree 2 or higher, use an appropriate method to solve.

Example

Solve the following system of equations using the elimination method.

$$3x - 2y = 7 \qquad (1)$$

$$5x + 4y = 1 \qquad (2)$$

Example

Solve the following system of equations using the elimination method.

$$3x - 2y = 7 \qquad (1)$$

$$5x + 4y = 1 \qquad (2)$$

Quadratic Equation

A quadratic equation in one variable is an equation in which x is the variable and has the general form

$$ax^2 + bx + c = 0$$

where a, b, and c are constants, and $a \neq 0$

$$x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Exercise

1. If x,y be positive real numbers satisfying the system of equations:

$2x + 3y = 11$ $\quad x - y = 1$ Find the value of: $x + y$

2. In a training workshop, there are two types of machined parts are steel parts and aluminum parts. Each steel part takes 20 minutes to machine and aluminum part takes 10 minutes to machine. A total of 30 parts are machined, and the total time used is 400 minutes. Find the number of steel parts and aluminum parts.

3. Solve the system of equations: $x + y = 6$ and $y = x^2 - 2x$

Exercise

4. In a training workshop, three types of parts are machined: steel, aluminum, and brass. In one day, a total of 32 parts are machined. The number of steel parts is 4 more than the number of aluminum parts, and the number of brass parts is 6 fewer than the number of steel parts. Define variables to represent the number of each type of part, set up a system of linear equations, and solve for the total of each.