Welcome to

FUNDAMENTAL MATHEMATICS

FOR INDUSTRY EDUCATION

Lesson 3 Part 3

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9641103 Fundamental Mathematics for Industry Education

Matrices and System of Linear Equations

Solving systems of equations with matrices means expressing them as $AX=B$ and using row operations to find the solution efficiently.

Methods for Solving Systems of Linear Equations Using Matrices

A system of linear equations can be expressed in matrix form as

$$AX=b$$

$$\begin{aligned} 3x + 2y - z &= 1 \\ x + 3y + z &= 2 \\ 2x - y + 2z &= 3 \end{aligned}$$

When

A is the coefficient matrix
X is the vector of unknown variables
b is the constant vector

- Inverse Matrix Method
- Gaussian Elimination
- Gauss–Jordan Elimination
- Cramer's Rule

Inverse Matrix Method

The inverse matrix method is based on the concept of matrix inverses. If the coefficient matrix A is a square matrix and its determinant is nonzero ($\det(A) \neq 0$), then A is invertible and the solution of the system can be obtained as

Conditions

The matrix A must be square and $\det(A) \neq 0$

Inverse Matrix Method

Procedure

Step 1: Write the system in matrix form $AX=b$

Step 2: Compute the determinant of A

Step 3: Compute the Minor Matrix $\rightarrow M=[M_{ij}]$

Step 4: Compute the Cofactor Matrix $\rightarrow C=[C_{ij}] \rightarrow C_{ij}=(-1)^{i+j}M_{ij}$

Step 5: Compute the Adjugate Matrix $\rightarrow \operatorname{adj}(A)=(\operatorname{cof}(A))^{T}$

Step 6: Compute the Inverse Matrix $\rightarrow A^{-1}=\dfrac{1}{\det(A)}\operatorname{adj}(A)$

Step 7: Solve $X=A^{-1}b$

Example

Determine the values of x, y, and z from the system of equations by inverse matrix method.

$$3x + 2y - z = 1$$
$$x + 3y + z = 2$$
$$2x - y + 2z = 3$$

Example

Step : 3 Compute the Minor Matrix $M = [M_{ij}]$

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Example

Step : 4 Compute the Cofactor Matrix $C = [C_{ij}]$ Step : 5 Compute the Adjugate Matrix

Example

Step : 6 Compute the Inverse Matrix

Gaussian Elimination

Gaussian elimination solves a system of linear equations by transforming the augmented matrix $[\ A|B\ ]$ using elementary row operations:

1. Row interchange
2. Row scaling by a nonzero constant
3. Row addition

The goal is to reduce the matrix to row echelon form $[\ A|B\ ]$, after which the solution can be obtained using back substitution.

Gaussian Elimination

Procedure

Step 1: Write the augmented matrix
Step 2: Eliminate entries below the first pivot
Step 3: Eliminate entries below the second pivot
Step 4: Back substitution

Example

Determine the values of x, y, and z from the system of equations by gaussian elimination.

$$3x+2y-z=1$$

$$x+3y+z=2$$

$$2x-y+2z=3$$

Step : 1 Write the augmented matrix

To simplify computations, swap Row 1 and Row 2 so the first pivot is 1:

Step : 2 Eliminate entries below the first pivot

Eliminate the 3 in Row 2 and eliminate the 2 in Row 3

Example

Step : 3 Eliminate entries below the second pivot

Step : 4 Back substitution

Gauss–Jordan Elimination

Gauss–Jordan elimination is an extension of Gaussian elimination. The augmented matrix $[\ A|B\ ]$ is further reduced to reduced row echelon form (RREF).

Key Characteristics

- Each leading entry is 1
- All other entries in the pivot columns are zero

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This method allows the solution to be read directly from the matrix without back substitution. However, it generally requires more computational steps than standard Gaussian elimination.

Gauss–Jordan Elimination

Procedure

Step 1: Write the augmented matrix
Step 2: Eliminate entries below the first pivot
Step 3: Make the second pivot equal to 1
Step 4: Eliminate the duplicate entry in column y
Step 5: Make the third pivot equal to 1
Step 6: above the third pivot
Step 7: Eliminate above the second pivot

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & b_1 \\ 0 & 1 & 0 & b_2 \\ 0 & 0 & 1 & b_3 \end{array}\right]$$

Example

Determine the values of x, y, and z from the system of equations by gauss–jordan Elimination.

Example

Step : 3 Make the second pivot equal to 1 Step : 4 Eliminate the duplicate entry in column y

Example

Step : 6 Eliminate above the third pivot

Step : 7 Eliminate above the second pivot

Cramer's Rule

Cramer's rule applies determinants to solve a system of nn linear equations in n variables. If $\det(A) \neq 0$, the solution is given by

$$x_i = \frac{\det(A_i)}{\det(A)}, i = 1, 2, 3, \ldots\ldots, n$$

where A_i is the matrix obtained by replacing the i column of A with the vector B

Limitations

Cramer's rule is practical only for small systems, as the computational cost of evaluating determinants increases rapidly with matrix size.

Cramer’s Rule

Procedure

Step 1: Write the system in matrix form $Ax = b$

Step 2: Compute the determinant of A

Step 3: Determinants D_x ,D_y , D_z

Step 3.1: Determinant D_x (replace the x column with b)

Step 3.2: Determinant D_y (replace the y column with b)

Step 3.3: Determinant D_z (replace the z column with b)

Step 4: Apply Cramer’s Rule

Example

Determine the values of x, y, and z from the system of equations by cramer's rule.

$$3x + 2y - z = 1$$
$$x + 3y + z = 2$$
$$2x - y + 2z = 3$$

Example

Step : 3 Determinants D_x, D_y, D_z

Example

Step : 3 Determinants D_x ,D_y ,D_z

Determinant D_y (replace the y column with b)

Example

Step : 3 Determinants D_x, D_y, D_z

Determinant D_z (replace the z column with b)

Example

Step : 4 Apply Cramer’s Rule

Exercise

1. Determine the values of x, y, and z from the system of equations by

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned}$$

1.1 Inverse Matrix Method
1.2 Gaussian Elimination
1.3 Gauss–Jordan Elimination
1.4 Cramer’s Rule

$x = 1, y = 2, z = 3$

Exercise

2. Determine the values of x, y, and z from the system of equations by

$$
\begin{aligned}
x + 2y - 3z &= 6 \\
2x - y + 4z &= 1 \\
x - y + z &= 3
\end{aligned}
$$

2.1 Inverse Matrix Method
2.2 Gaussian Elimination
2.3 Gauss–Jordan Elimination
2.4 Cramer's Rule

$$x = \frac{17}{5}, y = -\frac{7}{5}, z = -\frac{9}{5}$$

Exercise

3. Determine the values of x_1, x_2, and x_3 from the system of equations by

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 2 \\ 2x_1 - 2x_2 + 3x_3 &= 1 \\ x_1 + 2x_2 + 2x_3 &= -1 \end{aligned}$$

1.1 Inverse Matrix Method
1.2 Gaussian Elimination
1.3 Gauss–Jordan Elimination
1.4 Cramer's Rule

$x_1 = 5, x_2 = 0, x_3 = -3$

Exercise

4. Determine the values of x, y, and z from the system of equations by

$$
\begin{aligned}
-2y + x + 3z &= 9 \\
y + 3z &= 5 \\
5z + 2x - 5y &= 17
\end{aligned}
$$

1.1 Inverse Matrix Method
1.2 Gaussian Elimination
1.3 Gauss–Jordan Elimination
1.4 Cramer's Rule

$x = 1, y = -1, z = 2$