มหาวิทยาลัยราชภัฏนครปฐม
NAKHON PATHOM RAJABHAT UNIVERSITY

Welcome to

FUNDAMENTAL MATHEMATICS

FOR INDUSTRY EDUCATION

Lesson 05

Derivative of a Function Part 2

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9641103 Fundamental Mathematics for Industry Education

Higher-Order Derivatives

If $y = f(x)$ is a function that has a derivative at x, the derivative is

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

When the limit exists, $f'(x)$ is called the first derivative of $f(x)$.

If $f'(x)$ also has a derivative at x, the derivative of $f'(x)$ is called the second derivative of $f'(x)$ It is written as

$$\frac{d^2y}{dx^2} = f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

when the limit exists.

Higher-Order Derivatives

In general,

$$\frac{d^n y}{dx^n} = f^{(n)}(x) = \lim_{h \to 0} \frac{f^{(n-1)}(x+h) - f^{(n-1)}(x)}{h}$$

when the limit exists and n is a positive integer.

Order	Notation	Meaning
First	$f'(x), y'$	Rate of change
Second	$f''(x), y''$	Change of slope / curvature
Third	$f'''(x), y'''$	Derivative of the second derivative
(n)-th	$f^{(n)}(x), y^{(n)}$	Derivative taken (n) times

Example: Higher-Order Derivatives of Position

Let $s(t)$ represent the position or displacement of an object as a function of time t.

First Derivative is velocity.

$$v(t) = \frac{ds}{dt}$$

It represents the rate of change of position with respect to time.

Example: A car moving faster or slower.

Second Derivative is acceleration.

$$a(t) = \frac{d^2 s}{dt^2} = \frac{dv}{dt}$$

It represents the rate of change of velocity with respect to time.

Example:A car speeding up or braking.

Example: Higher-Order Derivatives of Position

Third Derivative is jerk.

$$j(t) = \frac{d^3 s}{dt^3} = \frac{da}{dt}$$

It represents the rate of change of acceleration.

Applications in engineering include:

- Elevator design
- High-speed trains
- Robotic arms
- to ensure **smooth motion without sudden changes.**

Example

$$s(t) = t^3 + 2t^2 + 5t$$

Velocity

Acceleration

Jerk

Example: Higher-Order Derivatives of Volumetric

Let $V(t)$ represent the volume of a fluid or substance as a function of time t.

First Derivative is volumetric flow rate.

$$Q(t) = \frac{dV}{dt}$$

Common units include:

m^3/s

L/min

Second Derivative is rate of change of the flow rate.

$$a(t) = \frac{d^2s}{dt^2} = \frac{dv}{dt}$$

It describes how quickly the flow rate is increasing or decreasing.

Applications include:

- accelerating fluid flow in pipes
- pump start-up or changing pump speed stran
- sient flow analysis

Example $f(x) = 7x^3 - 8x^2$ Find $f''(x)$

Sol

From

Example $f(x) = x^2\sqrt{x} - 5x$ Find $f''(x)$

Sol

From

Example $f(x) = \sin(2x)$ Find $f'''(x)$

Sol

From

Example $f(\alpha)=\sqrt[3]{2\alpha^3-5}$ Find $f''(\alpha)$

Sol

From

Example $f(\Delta)=\Delta^5+2\Delta^3-4\Delta^2-1$ Find $f^{(4)}(\Delta)$

Sol

From

Implicit Differentiation

Implicit differentiation is a technique used to find the derivative of a function when the variables x and y are given in an equation but y is not explicitly written as a function of x.

In other words, the equation is not in the form

$$y = f(x)$$

Example of an implicit equation

$$x^2 + y^2 = 25$$

Implicit Differentiation

Steps of Implicit Differentiation

Step 1: Differentiate both sides of the equation with respect to x

Step 2: Apply the chain rule to terms containing y

Step 3: Collect terms containing $\frac{dy}{dx}$ and solve the equation for $\frac{dy}{dx}$

Example $x^2 + y^2 = 25$ Find $f'(x)$

Sol

Example $4x^2 + 9y^2 = 36$ Find $f'(x)$

Sol

From $4x^2 + 9y^2 = 36$

Example $xy^2 - x + 16 = 0$ Find $f'(x)$

Sol

Example $2x^2 - 4xy + y^2 - 5x + 3y + 10 = 0$ Find $f'(x)$

Sol

Example $2x^2 - 4xy + y^2 - 5x + 3y + 10 = 0$ Find $f'(x)$

Sol

From