Welcome to

FUNDAMENTAL MATHEMATICS

FOR INDUSTRY EDUCATION

Lesson 06

Integration Part 1

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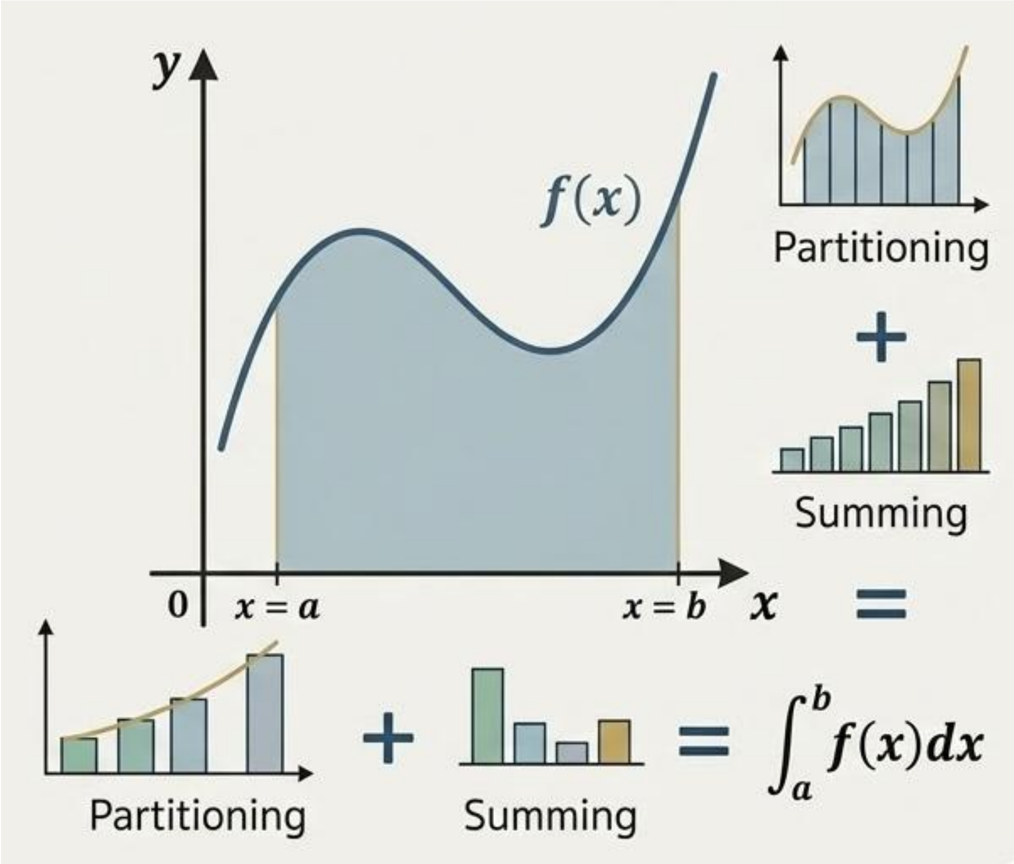
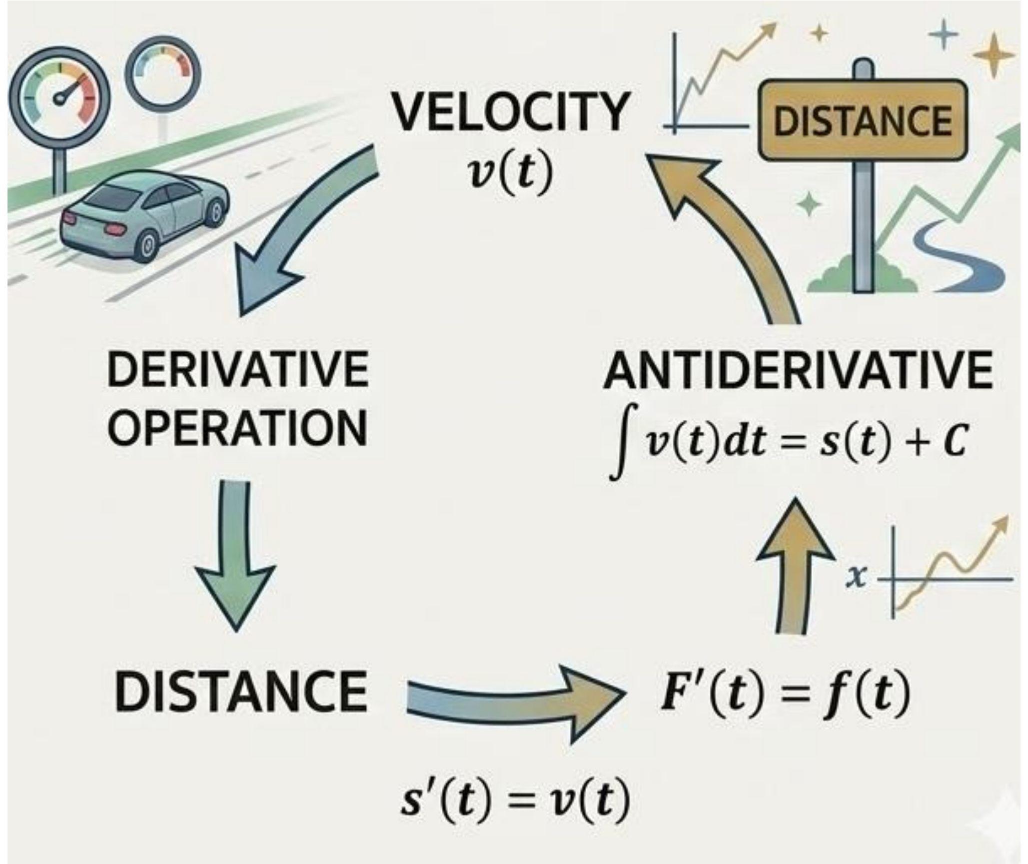
9641103 Fundamental Mathematics for Industry Education

What is an Integral?

Geometrically: It is the Area Under the Curve

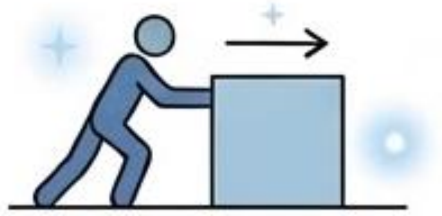
Computationally: It is the "Antiderivative." It is the reverse process of differentiation





What integration is it used for?

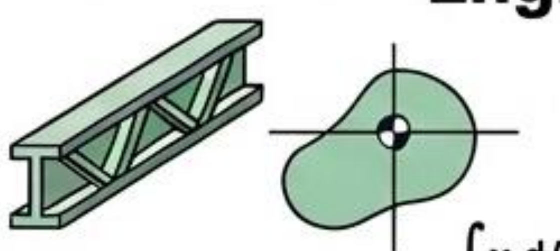
Physics and Mechanics



Work = $\int F\,dx$ Displacement = $\int v\,dt$

- Find **displacement** from velocity or velocity from acceleration.
- Calculate **'Work'** done by a variable force.

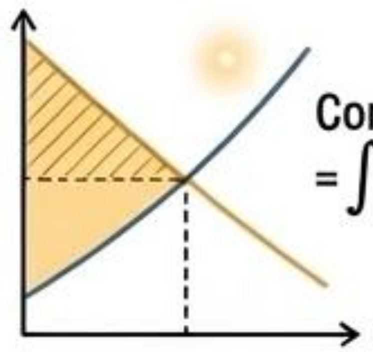
Engineering



Center of Gravity = $\dfrac{\int x\,dA}{\int dA}$ Moment of Inertia = $\int r^2\,dm$

- Calculate **Center of Gravity** for complex shapes.
- Find **Moment of Inertia.**

Economics

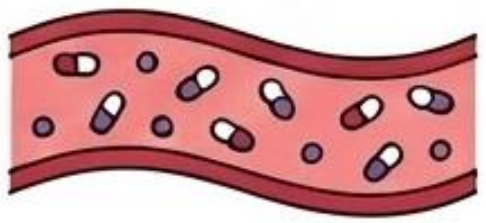


Consumer Surplus = $\int$(Demand - Price) dq

Growth = $\int$ rate dt

- Calculate **'Consumer Surplus'**.
- Predict **fund growth** over time.

Biology



Medicine concentration $\int c\,dt$ Microbial population $\int$ growth_rate dt

- Model **medicine** spread in the bloodstream.
- Model **microbial population growth rate.**

Indefinite Integrals

If a function $F(x)$ has a derivative equal to $f(x)$, that is

$$F'(x) = f(x)$$

then the indefinite integral of $f(x)$ is written as

$$\int f(x)\,dx = F(x) + C$$

Where
$\int$ = integral symbol
$f(x)$ = integrand (the function being integrated)
dx = variable of integration
$F(x)$ = antiderivative of $f(x)$
C = constant of integration

Indefinite Integrals

Why do we add the constant C

The derivative of any constant is zero.

Example:

$$\frac{d}{dx}x^2 = 2x$$

$$\frac{d}{dx}\left(x^2+5\right) = 2x$$

$$\frac{d}{dx}\left(x^2-99\right) = 2x$$

These functions all have the same derivative $2x$.

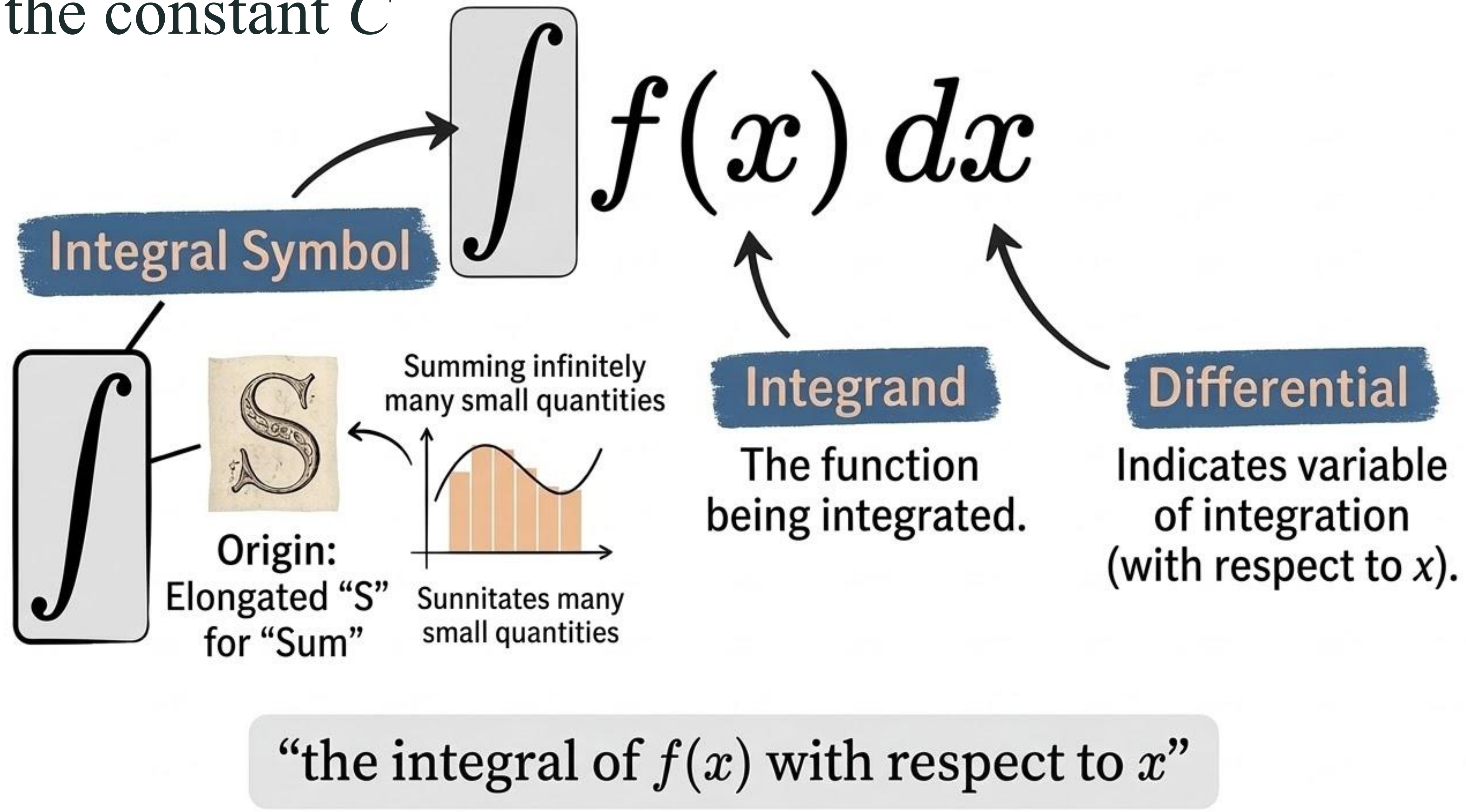
Therefore

$$\int 2x\,dx = x^2 + C$$

where C represents any real constant.

Indefinite Integrals

Why do we add the constant C



Indefinite Integrals

Let f and g be functions that have antiderivatives, and let k be a constant.

Then

$$\int f'(x)dx = f(x) + C$$

$$\int [f(x) + g(x)dx] = \int f(x)dx + \int g(x)dx$$

$$\int [f(x) - g(x)dx] = \int f(x)dx - \int g(x)dx$$

$$\int kf(x)dx = k\int f(x)dx$$

Integrals Formulas

1. $\int 1dx = x + C$

2. $\int x^n = \frac{x^{n+1}}{n+1} + C$

3. $\int \frac{1}{x} dx = \ln|x| + C$

4. $\int e^x = e^x + C$

5. $\int a^x dx = \frac{a^x}{\ln a} + C$

6. $\int \sin x dx = -\cos x + C$

7. $\int \cos x dx = \sin x + C$

8. $\int \sec x \tan x dx = \sec x + C$

9. $\int \sec^2 x dx = \tan x + C$

10. $\int \csc x \cot x dx = -\csc x + C$

11. $\int \csc^2 x dx = -\cot x + C$

12. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C = -\cot^{-1} x + C$

13. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C = -\cos^{-1} x + C$

14. $\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1} x + C = \csc^{-1} x + C$

Example $f'(x) = 3x^2 + x - 1$ Find $f(x)$

Sol

Example $\int \frac{2+x^2}{1+x^2}dx$

Sol

Example $\int \frac{(x-1)^2}{x^2} dx$

Sol

Integration by substitution

If $g(x)$ is a differentiable function and f is a continuous function on a given interval, then according to the substitution theorem, the following relationship holds.

Then $$\int f(g(x))g'(x) = \int f(u)du$$

Where $$u = g(x)$$

and from the differential relationship we obtain

$$du = g'(x)dx$$

Example $f'(x) = (2x+1)^{10}$ Find $f(x)$

Example $f'(x) = 5x(x^2 + 99)^3$ Find $f(x)$

Example $f'(x) = \cos(3x)$ Find $f(x)$

Indefinite Integrals

A definite integral represents the accumulated quantity of a function over a specific interval and geometrically corresponds to the net area under a curve between two points on the x-axis.

$$\int_a^b f(x)dx$$

Where

$\int$	=	integral symbol
$f(x)$	=	integrand (the function being integrated)
a	=	lower limit of integration
b	=	upper limit of integration
C	=	constant of integration

Indefinite Integrals

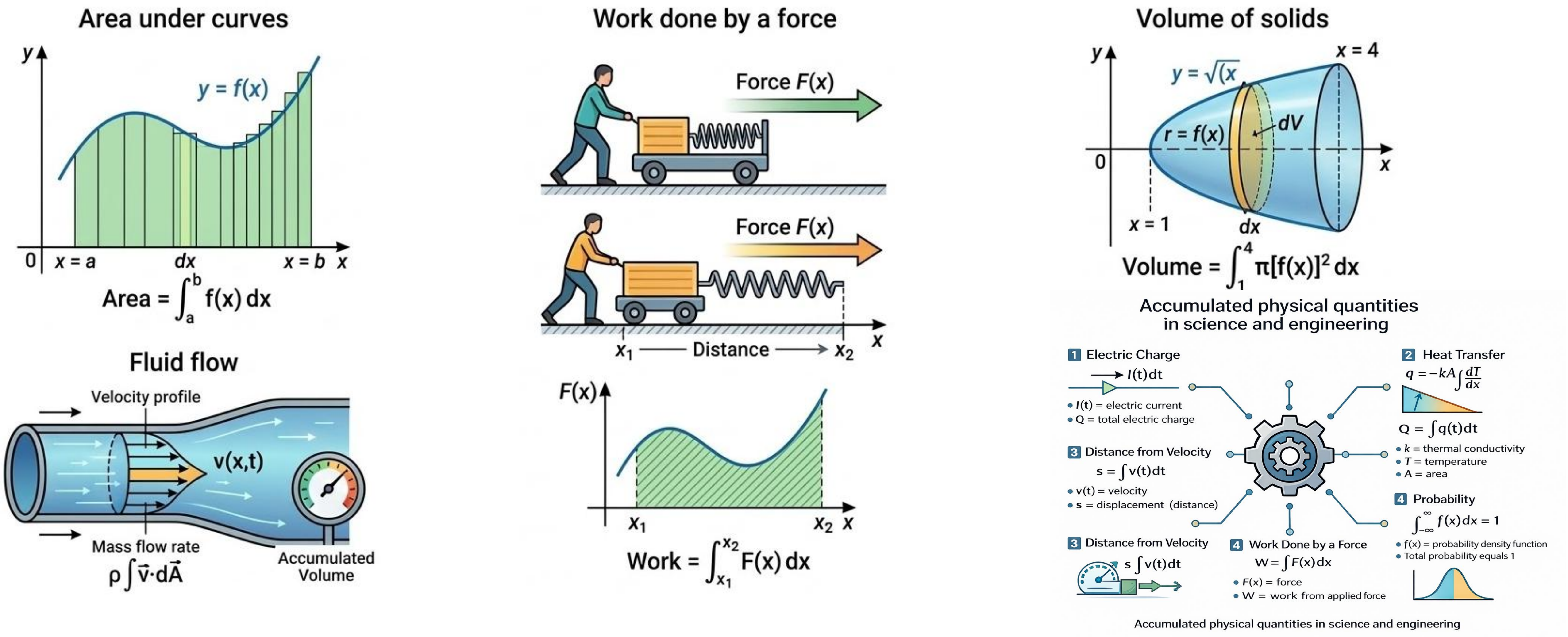
Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - f(a)$$

Procedure

Find the antiderivative of f(x).
Substitute the upper limit b and lower limit a.
Compute $F(b) - F(a)$.

Indefinite Integrals



Example $\int_0^1 3x^2 + x - 1dx$

Example $\int_1^2 (2x+3)^2\,dx$