มหาวิทยาลัยราชภัฏนครปฐม

NAKHON PATHOM RAJABHAT UNIVERSITY

Welcome to

FUNDAMENTAL MATHEMATICS

FOR INDUSTRY EDUCATION

Lesson 06

Integration Part 2

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9641103 Fundamental Mathematics for Industry Education

Integration by Parts

This principle is essentially the "Product Rule" of differentiation, but applied in reverse.

$$f(x) = x\sin x$$
$$f(x) = xe^{x}$$
$$f(x) = x(x-8)^{3}$$

The formula for Integration by Parts is:

$$\int u\,dv = uv - \int v\,du$$

Integration by Parts

Theorem and Formula

Let $u(x)$ and $v(x)$ be functions that are differentiable over a given interval. The formula for integration by parts is defined as: $\int udv = uv - \int vdu$

Derivation:

Starting from the Product Rule for differentiation

By integrating both sides with respect to x

Rearranging the equation to solve for $\int udv$ yields the standard formula shown above:

Integration by Parts

How to Choose *u* and *dv*

Selection Strategy: The LIATE Rule

- L (Logarithmic functions): $\ln x$
- I (Inverse trigonometric functions): $\arctan x$
- A (Algebraic functions): x^n
- T (Trigonometric functions): $\sin x$, $\cos x$
- E (Exponential functions): e^x

Example $\int xe^{x}dx$

Example $\int x\sin(x)dx$

Example $\int x \ln x dx$

Example $\int x(x+5)^{10}\,dx$

Step 1 Choose u and dv

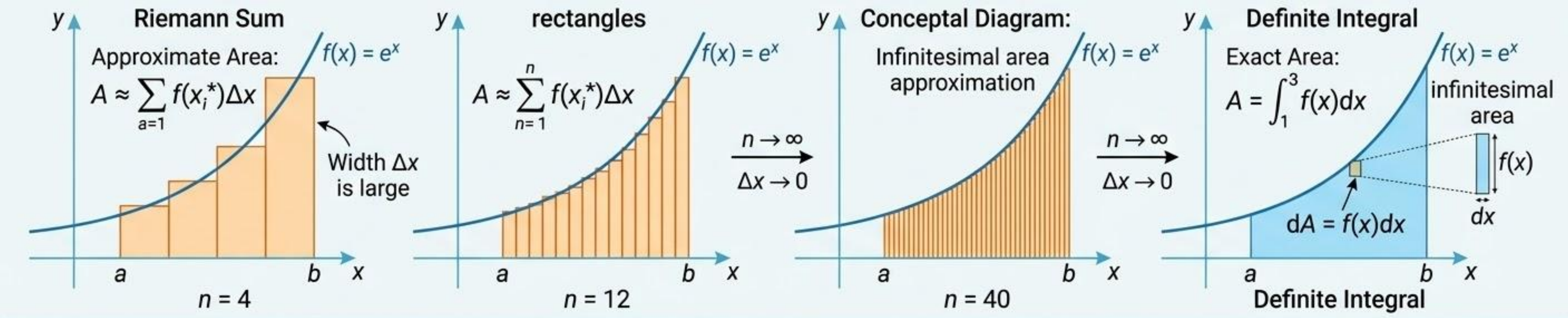
Example $\int x\sqrt{x+1}dx$

Example $\int_0^1 x(x+1)^3 dx$

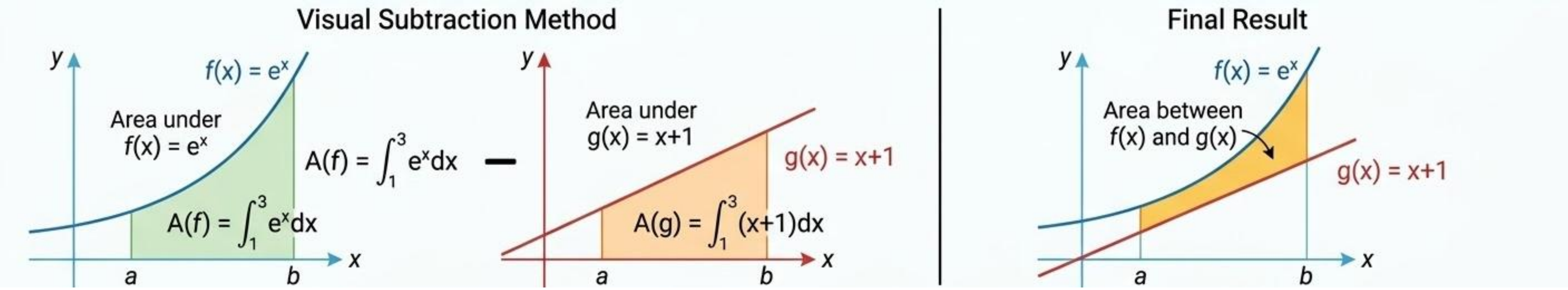
Example $\int_0^1 x(x+1)^3 dx$

Applications of Integration

Area Under a Curve



Area Between Curves



Applications of Integration

Area Under a Curve

If a function $f(x)$ is continuous on the interval $[a,b]$ the area under the curve is given by

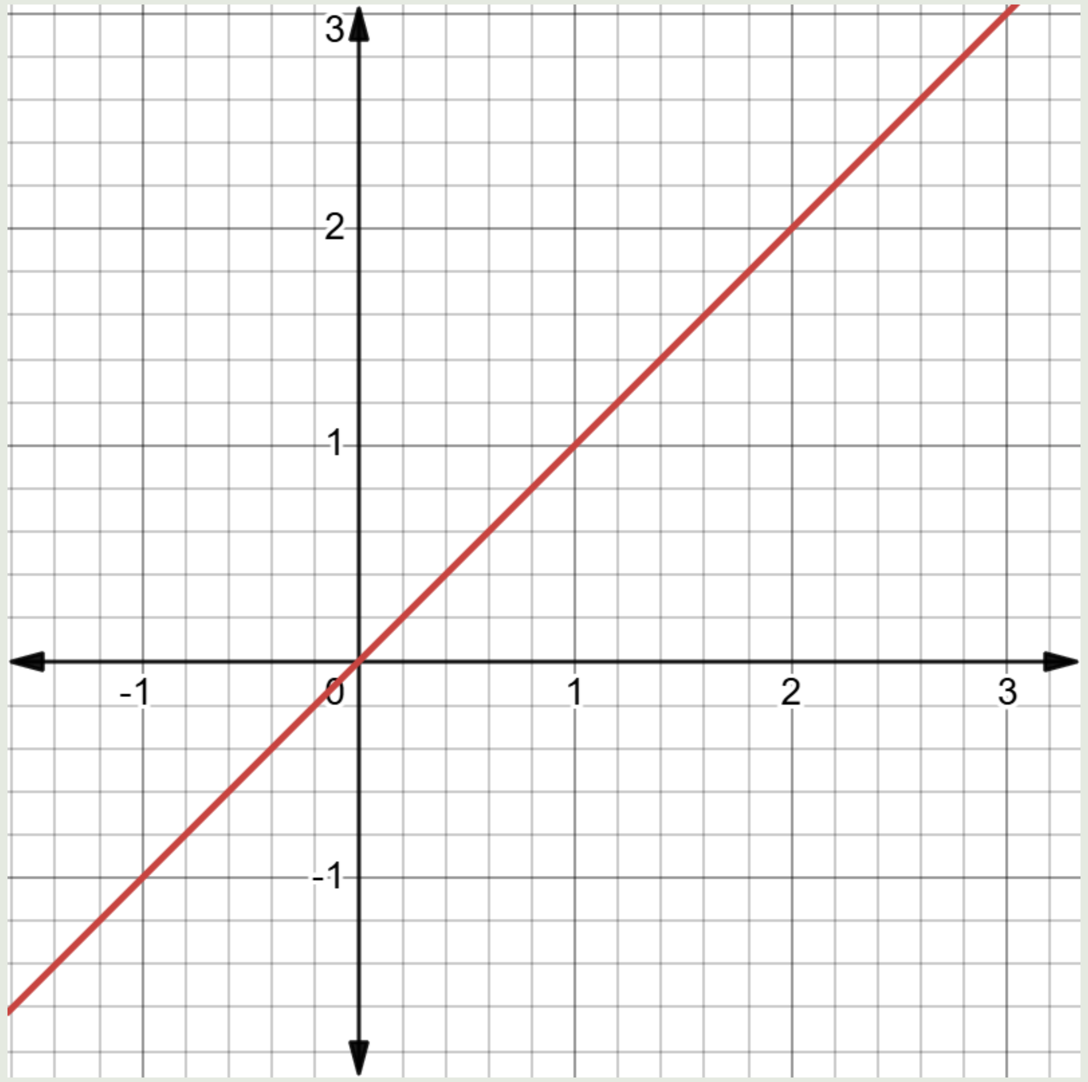
$$A = \int_a^b f(x)\,dx$$

Where

A	= Aera
$f(x)$	= the given function
a	= the lower limit (left boundary)
b	= the upper limit (right boundary)
dx	= an infinitesimally small change in x

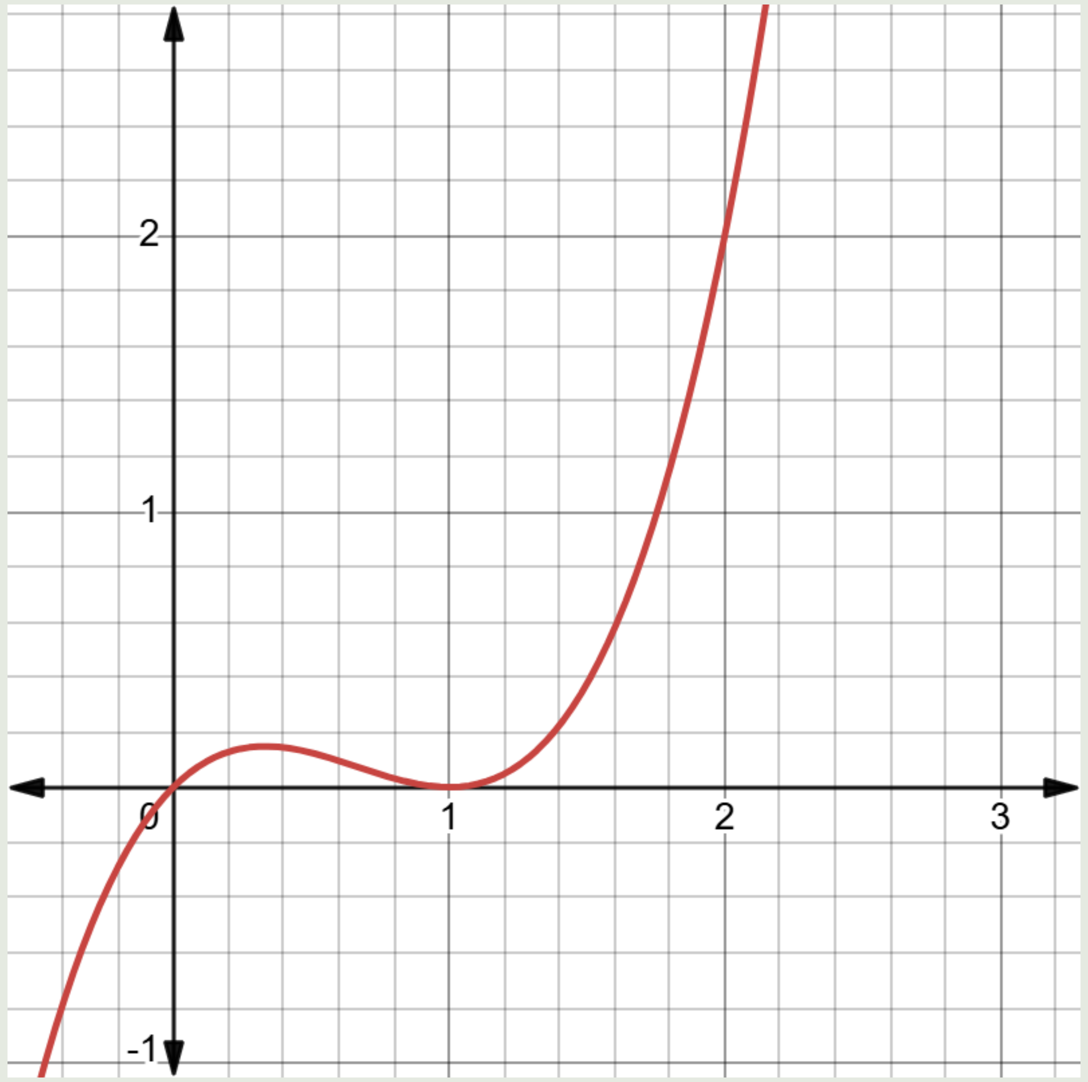
Example

Find the area under the curve $f(x)=x$ from $x=0$ to $x=2$.



Example

Find the area under the curve $f(x)=x^3-2x^2+x$ from $x=1$ to $x=2$.



Applications of Integration

Area Between Curves

The area between curves refers to the region enclosed between the graphs of two functions. This area can be calculated using the concept of the definite integral.

Suppose

$y = f(x)$ is the upper curve,

$y = g(x)$ is the lower curve,

$$A = \int_a^b \left[f(x) - g(x) \right] dx$$

Where

A	= Aera
$f(x)$	= the upper function
$g(x)$	= the lower function
a *and* b	= the intersection points or the limits of integration

Example

Find the area under the curve $f(x) = x+1$ $g(x) = x^2+x$ from $x = 0$ to $x = 1$.