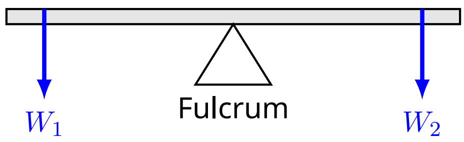
Physics Instructional Material

Unit 4: Mechanical Equilibrium

Fundamental Principles of Statics and Rigid Body Dynamics

Statics in Balance



Synthesized for Academic Excellence
Based on the National Physics Curriculum Guidelines

Prepared by: Your Physics Instructor

Contents

1 Course Objectives

After completing this unit, students will be able to:

- Explain the conditions for mechanical equilibrium of objects.
- Define and calculate the moment of a force (torque) and its effects on rotation.
- Analyze the concept of a couple and its resulting moment.
- Construct accurate Free Body Diagrams (FBD) for objects in equilibrium.
- Differentiate between Center of Mass (CM) and Center of Gravity (CG).
- Evaluate the stability of objects based on their base of support and CG location.

2 The Nature of Mechanical Equilibrium

In physics, **Mechanical Equilibrium** is a state where an object experiences no change in its motion. This includes both translational and rotational motion. An object in mechanical equilibrium must satisfy two conditions regarding the net force and net torque acting upon it.

2.1 Types of Equilibrium

Equilibrium can be classified into two main categories:

1. **Static Equilibrium:** The object is at rest ($v = 0$ and $\omega = 0$).
2. **Dynamic Equilibrium:** The object moves with a constant velocity ($v =$ constant) or rotates with a constant angular velocity ($\omega =$ constant).

3 Conditions for Mechanical Equilibrium

For a rigid body to be in total mechanical equilibrium, it must satisfy the following mathematical conditions:

3.1 Translational Equilibrium (The First Condition)

The vector sum of all external forces acting on the object must be zero.

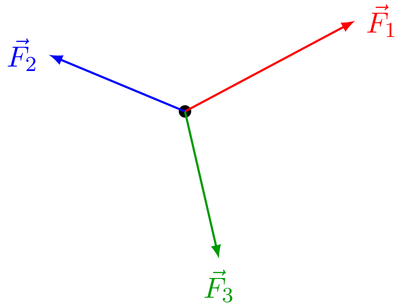
$$\sum \vec{F} = 0 \quad \Longrightarrow \quad \sum F_x = 0, \quad \sum F_y = 0 \tag{1}$$

3.2 Rotational Equilibrium (The Second Condition)

The vector sum of all external torques (moments) acting on the object about any axis must be zero.

$$\sum \vec{\tau} = 0 \quad \text{or} \quad \sum M = 0 \tag{2}$$

Equilibrium of Concurrent Forces



4 Moment of a Force (Torque)

The **Moment of a Force** (often called Torque, τ) is a measure of the tendency of a force to cause an object to rotate about a specific point or axis.

4.1 Mathematical Definition

The magnitude of the moment is defined as the product of the force magnitude and the perpendicular distance (lever arm) from the axis of rotation to the line of action of the force.

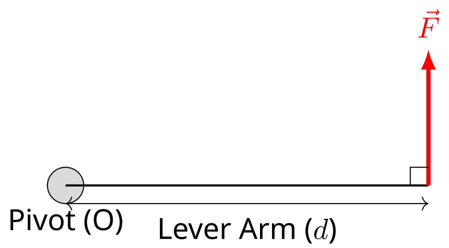
$$M = F \times d \tag{3}$$

Where:

- M is the Moment (measured in Newton-meters, $N \cdot m$).
- F is the applied Force (N).
- d is the perpendicular distance (lever arm) from the pivot (m).

4.2 Direction and Sign Convention

- **Counter-Clockwise (CCW):** Conventionally treated as positive ($+$).
- **Clockwise (CW):** Conventionally treated as negative ($-$).



4.3 Varignon's Theorem

The theorem states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

$$M_O = (F \sin\theta) \cdot r \tag{4}$$

5 Force Couples

A **Couple** consists of two parallel forces that are equal in magnitude, opposite in direction, and do not share the same line of action.

5.1 Properties of a Couple

- A couple produces purely rotational motion (no translation).
- The net force of a couple is zero ($\sum F = 0$).
- The moment of a couple is the same about any point in its plane.

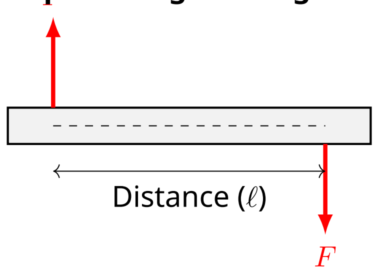
5.2 Moment of a Couple Calculation

The magnitude of the moment of a couple (M_c) is:

$$M_c = F \times \ell \quad (5)$$

where ℓ is the perpendicular distance between the two forces.

A Couple acting on a Rigid Body



6 Free Body Diagrams in Equilibrium

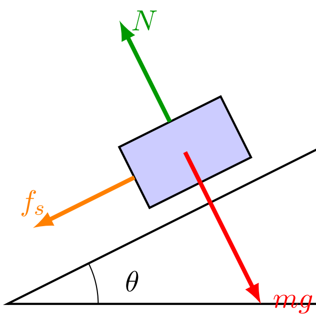
A **Free Body Diagram (FBD)** is a graphical illustration used to visualize the applied forces, moments, and resulting reactions on a body in a given condition.

6.1 Steps to Draw an FBD

1. Identify the object or system to be analyzed.
2. "Isolate" the object from its environment.
3. Draw vectors representing all external forces (Weight, Normal force, Tension, Friction).
4. Label all forces with their magnitudes and directions.
5. Establish a coordinate system (usually x and y).

6.2 Example: A Block on an Inclined Plane

Consider a block of mass m at rest on a plane inclined at an angle θ.



7 Center of Mass and Center of Gravity

7.1 Center of Mass (CM)

The **Center of Mass** is the unique point where the entire mass of an object may be considered to be concentrated for the purpose of describing its translational motion.

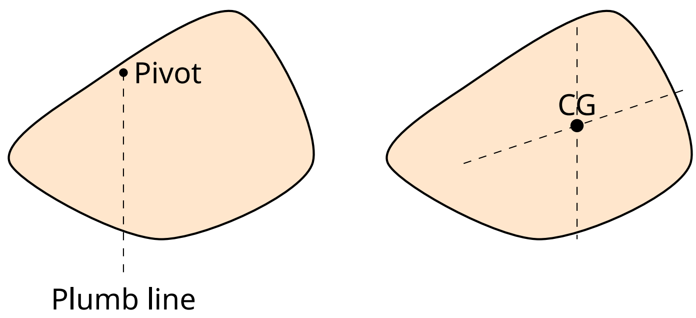
$$X_{cm} = \frac{\sum m_i x_i}{\sum m_i} \tag{6}$$

7.2 Center of Gravity (CG)

The **Center of Gravity** is the point through which the resultant force of gravity (weight) acts on an object. In a uniform gravitational field, the Center of Gravity coincides with the Center of Mass.

7.3 Experimental Determination

For irregular flat objects, the CG can be found by suspending the object from different points and marking the vertical plumb lines. The intersection of these lines is the CG.

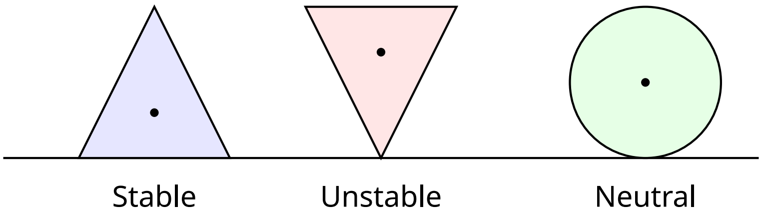


8 Stability of Objects

Stability refers to the ability of an object to return to its original position after being slightly displaced. It is determined by the position of the CG relative to the **Base of Support**.

8.1 Three States of Stability

1. **Stable Equilibrium:** A small displacement raises the CG. The object returns to its original position (e.g., a cone on its base).
2. **Unstable Equilibrium:** A small displacement lowers the CG. The object moves further away from its original position (e.g., a cone balanced on its tip).
3. **Neutral Equilibrium:** A displacement does not change the height of the CG (e.g., a cylinder or ball rolling on a horizontal surface).



8.2 Factors Affecting Stability

- **Height of CG:** Lowering the CG increases stability.
- **Area of Base:** Increasing the base of support increases stability.

9 Problem Solving Strategies

When solving equilibrium problems, follow this structured approach:

1. **Sketch the Situation:** Draw a simple diagram of the physical setup.
2. **Select the System:** Choose which object(s) to analyze.
3. **Draw an FBD:** Include all forces (Weight, Normal, Tension, etc.).
4. **Choose Coordinate Axes:** Usually x (horizontal) and y (vertical).
5. **Apply $\sum F = 0$:** Break forces into components.
   - $\sum F_x = 0$
   - $\sum F_y = 0$
6. **Apply $\sum M = 0$:** Choose a pivot point that eliminates unknown forces (usually where an unknown force acts).
7. **Solve Equations:** Use algebra to find the target quantities.

10 Worked Example: The Uniform Beam

A uniform beam of length $4.0\ m$ and mass $20\ kg$ is supported by two scales at its ends. A $50\ kg$ block is placed $1.0\ m$ from the left end. Find the readings on both scales.

10.1 Step 1: Identify Forces

- Weight of beam (W_b): $20\ kg \times 9.8 = 196\ N$ (acts at center, $2.0\ m$).
- Weight of block (W_{block}): $50\ kg \times 9.8 = 490\ N$ (acts at $1.0\ m$).
- Reaction forces R_1 (left) and R_2 (right).

10.2 Step 2: Apply Rotational Equilibrium (Pivot at Left End)

$\sum M_{left} = 0$:

$$(490 \cdot 1.0) + (196 \cdot 2.0) - (R_2 \cdot 4.0) = 0 \tag{7}$$

$$490 + 392 = 4 \cdot R_2 \implies R_2 = 220.5\ N \tag{8}$$

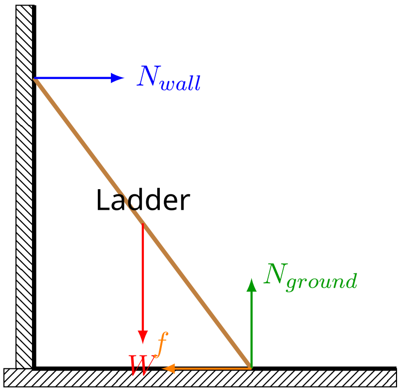
10.3 Step 3: Apply Translational Equilibrium

$\sum F_y = 0$:

$$R_1 + R_2 - 490 - 196 = 0 \implies R_1 = 686 - 220.5 = 465.5\ N \tag{9}$$

11 Review Exercises

1. **Conceptual:** Why do racing cars have a very low profile and wide tires? Explain in terms of stability.
2. **Calculation:** A $10\ kg$ mass is suspended by two strings. String A makes an angle of $30°$ with the vertical, and String B makes an angle of $60°$ with the vertical. Calculate the tension in each string.
3. **Moment:** A wrench is $25\ cm$ long. If a mechanic applies a force of $50\ N$ at an angle of $70°$ to the wrench handle, what is the torque applied to the bolt?
4. **Rigid Body:** A uniform ladder of length $5\ m$ and mass $15\ kg$ leans against a smooth vertical wall. The base of the ladder is $3\ m$ from the wall. Calculate the normal force from the wall and the friction force from the ground.



— End of Unit 4 Instructional Material —