Physics Instructional Material

Unit 5: Work and Energy

Fundamental Principles of Mechanics and Energetics

Course Objectives

- Analyze and calculate work done by constant and varying forces.
- Define and compute Kinetic and Potential Energy.
- Apply the Work-Energy Theorem to physical systems.
- Explain the Law of Conservation of Mechanical Energy.
- Evaluate the efficiency and mechanical advantage of simple machines.

Synthesized for Academic Excellence
Based on the Grade 10 National Physics Curriculum

Contents

1 Introduction to Work and Energy

In physics, the concepts of **work** and **energy** provide an alternative framework to Newton's Laws for analyzing the motion of objects. While Newton's Laws focus on force and acceleration, the energy approach focuses on the scalar quantities that govern changes in state.

2 5.1 Work Done by a Constant Force

Work is done when a force acting on an object causes a displacement. Formally, if a constant force $\vec{F}$ acts on an object while it undergoes a displacement $\Delta\vec{x}$, the work W done by the force is defined as the product of the magnitude of the displacement and the component of the force in the direction of displacement.

2.1 Mathematical Definition

The standard equation for work is:

$$W = F\Delta x \cos\theta \tag{1}$$

Where:

- W is the work (measured in Joules, J).
- F is the magnitude of the constant force (Newtons, N).
- Δx is the magnitude of displacement (meters, m).
- θ is the angle between the force vector and the displacement vector.

2.2 Vector Representation of Work

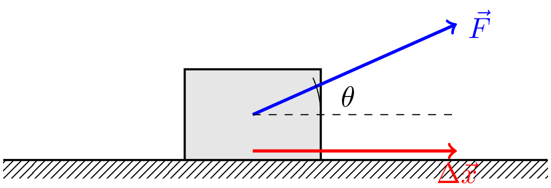


Figure 1.1: Work done by a force at an angle.

2.3 Signs of Work

The value of $\cos\theta$ determines the sign of the work done:

1. **Positive Work ($0° \le \theta < 90°$):** The force has a component in the same direction as the displacement (e.g., pulling a sled).
2. **Zero Work ($\theta = 90°$):** The force is perpendicular to the displacement. No energy is transferred (e.g., the normal force on a sliding block).
3. **Negative Work ($90° < \theta \le 180°$):** The force opposes the motion (e.g., friction).

3 5.2 Work Done by a Varying Force

In many real-world scenarios, such as stretching a spring, the force is not constant. In these cases, we cannot simply use $F\Delta x$. Instead, we analyze the relationship via a Force-Position graph.

3.1 Graphical Analysis

The work done by a force as an object moves from position x_i to x_f is equal to the **area under the Force vs. Position curve**.

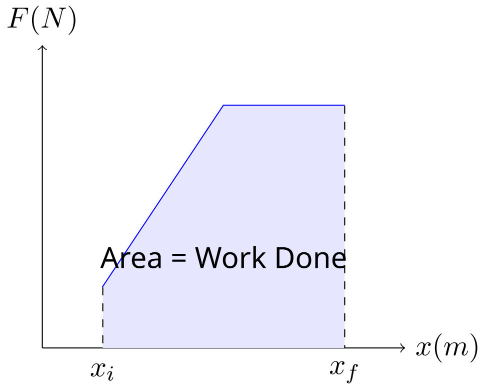


Figure 2.1: Work as area under the curve.

For a linear varying force (like a spring), the area forms a trapezoid or triangle:

$$W = \bar{F}\Delta x = \frac{F_i + F_f}{2}\Delta x \tag{2}$$

4 5.3 Power

Power is defined as the rate at which work is done or the rate of energy transfer.

4.1 Average Power

The average power P_{av} is the total work W divided by the time interval Δt:

$$P_{av} = \frac{W}{\Delta t} \tag{3}$$

The SI unit for power is the **Watt (W)**, where $1W = 1J/s$.

4.2 Instantaneous Power

For an object moving at a constant velocity v under a force F:

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta \tag{4}$$

5 5.4 Mechanical Energy

Energy is the capacity to do work. In mechanics, we categorize it into Kinetic Energy and Potential Energy.

5.1 5.4.1 Kinetic Energy (E_k)

Kinetic energy is the energy possessed by an object due to its motion.

$$E_k = \frac{1}{2}mv^2 \tag{5}$$

Where m is mass and v is speed. Because v is squared, kinetic energy is always non-negative.

5.2 The Work-Energy Theorem

One of the most powerful theorems in physics states that the net work done by all forces acting on an object is equal to the change in its kinetic energy:

$$W_{\text{net}} = \Delta E_k = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \tag{6}$$

5.3 5.4.2 Potential Energy (E_p)

Potential energy is "stored" energy due to the configuration or position of an object within a field.

5.3.1 Gravitational Potential Energy

When an object of mass m is at a height h above a reference level:

$$E_p = mgh \tag{7}$$

The change in potential energy depends only on the vertical height difference, not the path taken.

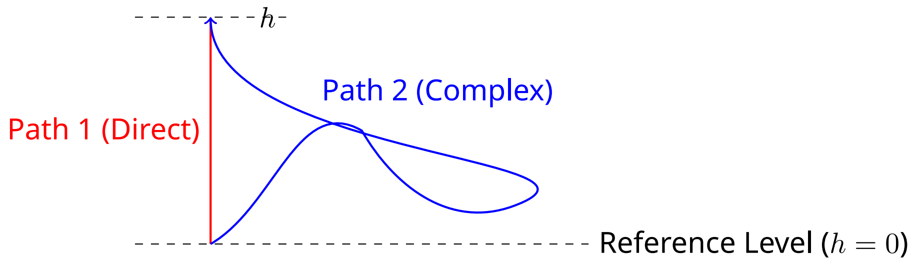


Figure 4.1: Path independence of gravitational work.

5.3.2 Elastic Potential Energy

Energy stored in a deformed elastic object (like a spring) is:

$$E_p = \frac{1}{2}kx^2 \tag{8}$$

Where k is the spring constant (N/m) and x is the displacement from equilibrium.

6 5.5 Law of Conservation of Mechanical Energy

Mechanical energy E_{mech} is the sum of kinetic and potential energy:

$$E_{\text{mech}} = E_k + E_p \tag{9}$$

6.1 Conservative Forces

A force is **conservative** if the work it does on an object moving between two points is independent of the path (e.g., gravity, spring force). If only conservative forces do work, the total mechanical energy remains constant:

$$E_{ki} + E_{pi} = E_{kf} + E_{pf} \tag{10}$$

6.2 Non-Conservative Forces

Forces like friction and air resistance are **non-conservative**. They transform mechanical energy into thermal energy. In such cases:

$$W_{\text{nc}} = \Delta E_{\text{mech}} = (E_{kf} + E_{pf}) - (E_{ki} + E_{pi}) \tag{11}$$

6.3 Case Study: Pendulum Energy Transformation

A swinging pendulum constantly converts energy between potential and kinetic forms. At the highest point, speed is zero ($E_k = 0, E_p =$ max). At the lowest point, height is minimal ($E_p = 0, E_k =$ max).

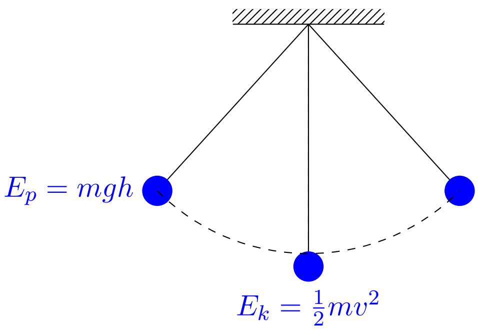


Figure 5.1: Energy conversion in a pendulum.

7 5.6 Simple Machines

Simple machines are devices that change the magnitude or direction of a force to make work easier. They do **not** reduce the amount of work required; they leverage the trade-off between force and distance.

7.1 Key Definitions

- **Work Input (W_{in}):** Work done *on* the machine ($F_{in} \times s_{in}$).
- **Work Output (W_{out}):** Work done *by* the machine ($F_{out} \times s_{out}$).
- **Mechanical Advantage (MA):** Ratio of output force to input force.
- **Efficiency (η):** Percentage of input work converted to useful output work.

7.2 Mechanical Advantage (MA)

There are two types of MA:

1. **Actual Mechanical Advantage (AMA):** Considers friction.

$$AMA = \frac{F_{\text{out}}}{F_{\text{in}}} \tag{12}$$

2. **Ideal Mechanical Advantage (IMA):** Assumes no friction.

$$IMA = \frac{s_{\text{in}}}{s_{\text{out}}} \tag{13}$$

7.3 Efficiency (η)

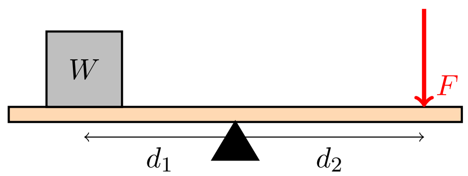
Efficiency is calculated as:

$$\eta = \frac{W_{\text{out}}}{W_{\text{in}}} \times 100\% = \frac{AMA}{IMA} \times 100\% \tag{14}$$

7.4 Types of Simple Machines

7.4.1 The Lever

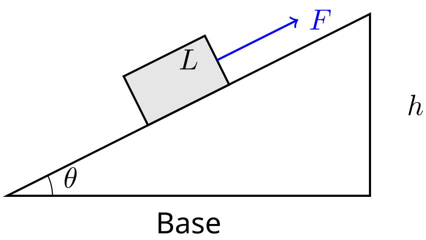
Based on the principle of moments (torques).



For an ideal lever: $F \times d_2 = W \times d_1 \implies IMA = \frac{d_2}{d_1}$.

7.4.2 The Inclined Plane

An inclined plane allows a heavy load to be lifted to a height h using a smaller force F over a longer distance L.



The Ideal Mechanical Advantage is:

$$IMA = \frac{L}{h} = \frac{1}{\sin\theta} \tag{15}$$

7.4.3 The Pulley System

A single fixed pulley only changes direction ($IMA = 1$). A single movable pulley doubles the force ($IMA = 2$). In a system of multiple pulleys, the IMA is equal to the number of rope segments supporting the movable load.

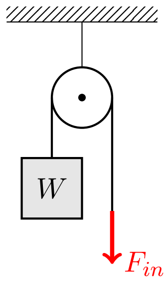


Figure 6.1: Single fixed pulley.

8 Worked Examples for Practical Mastery

8.1 Example 1: Work at an Angle

A worker pulls a crate with a force of $124N$ at an angle of 30° to the horizontal. If the crate is moved $500m$ along the floor, calculate the work done.

Solution: Given: $F = 124N$, $\theta = 30^\circ$, $\Delta x = 500m$. Using $W = F\Delta x \cos\theta$:

$$W = (124N)(500m)\cos(30^\circ)$$
$$W = 62,000 \times \frac{\sqrt{3}}{2}$$
$$W \approx 53,692J = 5.37 \times 10^4 J$$

8.2 Example 2: Conservation of Energy

A $0.1kg$ fruit falls from a height of $5m$. Calculate its kinetic energy when it has fallen halfway.

Solution: Initial State (Top): $E_k = 0, E_p = mgh = (0.1)(9.8)(5) = 4.9J$. Total Energy $E = 4.9J$. At Halfway ($h = 2.5m$): $E_p = mgh = (0.1)(9.8)(2.5) = 2.45J$. By Conservation of Energy: $E = E_k + E_p \implies 4.9 = E_k + 2.45$. $E_k = 2.45J$.

8.3 Example 3: Efficiency of an Inclined Plane

A $25kg$ object is pushed up a $5m$ long ramp to a height of $1m$. A force of $80N$ is applied parallel to the ramp. Find the efficiency.

Solution: $W_{in} = F \times L = (80N)(5m) = 400J$. $W_{out} = mgh = (25kg)(9.8m/s^2)(1m) = 245J$. $\eta = \frac{W_{out}}{W_{in}} \times 100\% = \frac{245}{400} \times 100\% = 61.25\%$.

9 Review Exercises

1. **Conceptual:** Can a centripetal force doing work on an object in circular motion? Explain why or why not.
2. **Calculation:** A motor with a power of $1000W$ is used to lift a $50kg$ mass. How high can it lift the mass in 2 minutes, assuming 100% efficiency?
3. **Graphical:** Draw a Force vs. Position graph for a spring where $k = 200N/m$. Calculate the work done to stretch it from 0 to $0.1m$ using the area.
4. **Energy:** A $4kg$ ball is swinging on a $2m$ string. At the lowest point, it has a speed of $6m/s$. What is its speed when the string makes a $60°$ angle with the vertical?
5. **Simple Machines:** A pulley system has 4 support ropes. If an input force of $50N$ lifts a $150N$ load:
   - What is the IMA?
   - What is the AMA?
   - What is the efficiency?

10 Answer Key

1. **No.** Centripetal force is always perpendicular to the velocity (displacement) vector. Since $\theta = 90°$, $\cos 90° = 0$, resulting in zero work.
2. **Height = 245 meters.** $P = \frac{W}{t} \implies W = P \times t = 1000W \times 120s = 120,000J$. $W = mgh \implies 120,000 = (50)(9.8)h \implies h \approx 244.9m$.
3. **Work = 1 Joule.** Area of triangle $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 0.1 \times (200 \times 0.1) = 1J$.
4. **Speed** $\approx 4.05m/s$**.** Using conservation of energy: $\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + mgh$. $h = L(1 - \cos 60°) = 2(1 - 0.5) = 1m$. $18 = \frac{1}{2}v_2^2 + 9.8(1) \implies \frac{1}{2}v_2^2 = 8.2 \implies v_2 = \sqrt{16.4} \approx 4.05m/s$.
5. **IMA = 4, AMA = 3, Efficiency = 75%.**

— End of Unit 5 Course Materials —