Physics Instructional Material

Unit 2: Momentum and Collisions

Fundamental Analysis of Motion and Interaction

Course Objectives

After completing this unit, students will be able to:

- Explain the concept of momentum and calculate its value for various objects.
- Analyze the relationship between force and the rate of change of momentum.
- Define impulse and determine it from force-time graphs.
- Apply the Law of Conservation of Momentum to solve collision and explosion problems.
- Differentiate between elastic and inelastic collisions in one dimension.

Synthesized for Academic Excellence
Based on Advanced Physics Curriculum Standards

Contents

1 Linear Momentum

In our daily lives, we intuitively understand that a heavy truck moving at high speed is much harder to stop than a small bicycle moving slowly. This ”quantity of motion” is what physicists define as **Momentum**.

1.1 Definition and Mathematical Expression

Linear momentum is a physical quantity that describes the motion of an object as the product of its mass and its velocity. It represents the tendency of an object to continue moving in its current direction.

Definition: Linear Momentum

The momentum $\vec{p}$ of an object is defined as:

$$\vec{p} = m\vec{v} \tag{1}$$

Where:

- m is the mass of the object (kg)
- $\vec{v}$ is the velocity of the object (m/s)

1.2 Properties of Momentum

1. **Vector Quantity:** Momentum has both magnitude and direction. The direction of the momentum vector is always the same as the direction of the velocity vector.
2. **SI Units:** The unit of momentum is the kilogram-meter per second (kg · m/s).
3. **Dependence:** Momentum depends linearly on both mass and velocity. Doubling the mass or velocity of an object doubles its momentum.

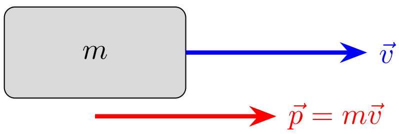


Figure 1.1: Vector relationship of momentum

1.3 Worked Example 1.1

Question: A bird with a mass of 0.030 kg is flying north at a speed of 8.0 m/s. Calculate the magnitude of its momentum.

Solution: Given: $m = 0.030$ kg, $v = 8.0$ m/s.

$$p = m \times v$$
$$p = 0.030 \text{ kg} \times 8.0 \text{ m/s}$$
$$p = 0.24 \text{ kg} \cdot \text{m/s}$$

The magnitude of the bird's momentum is 0.24 kg · m/s toward the north.

2 Force and the Change in Momentum

Sir Isaac Newton originally formulated his Second Law of Motion in terms of momentum rather than acceleration. This perspective is vital for understanding systems where forces act over a period of time.

2.1 Newton's Second Law Revisited

Newton stated that the net force acting on an object is equal to the rate of change of its momentum.

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t} \tag{2}$$

If the mass remains constant, we can derive the familiar $F = ma$:

$$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = m\left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right) = m\vec{a}$$

2.2 The Concept of Impulse

When a force acts on an object for a short time interval Δt (like a bat hitting a ball), it causes a change in momentum. This product of force and time is called **Impulse** ($\vec{I}$).

Impulse-Momentum Theorem

The impulse $\vec{I}$ is defined as:

$$\vec{I} = \vec{F}_{net}\Delta t = \Delta \vec{p} \tag{3}$$

Impulse is equal to the change in momentum of the object. Its SI unit is the Newton-second (N · s).

2.3 Graphical Interpretation of Impulse

In many real-world situations, the force acting on an object is not constant. For example, during a golf swing, the force increases as the ball compresses and decreases as it leaves the clubface. Impulse can be determined by the **area under the force-time graph**.

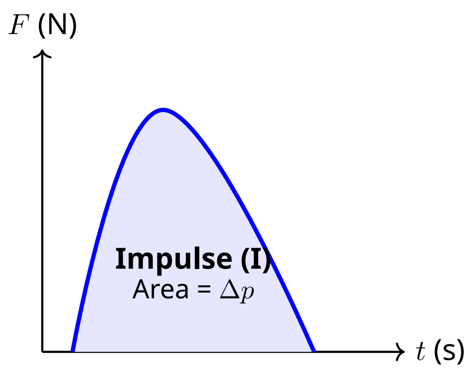


Figure 2.1: Force-Time graph showing Impulse

2.4 Safety Applications: Increasing Collision Time

Understanding impulse is crucial for safety design. To reduce the average force $\vec{F}$ felt by a passenger during a crash, we must increase the time Δt over which the momentum change occurs.

$$\vec{F}_{avg} = \frac{\Delta \vec{p}}{\uparrow \Delta t} \implies \downarrow \vec{F}_{avg}$$

This is the scientific basis for **airbags**, **crumple zones**, and **padded helmets**.

3 Conservation of Momentum

One of the most powerful laws in physics is the Law of Conservation of Momentum. It allows us to predict the outcomes of interactions without knowing the details of the forces involved.

3.1 The Principle of Conservation

In an **isolated system** (a system where the net external force is zero), the total momentum of the system remains constant over time.

Law of Conservation of Momentum

For an isolated system:

$$\sum \vec{p}_{initial} = \sum \vec{p}_{final} \tag{4}$$

Or for two objects m_1 and m_2:

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2 \tag{5}$$

Where $\vec{u}$ represents initial velocities and $\vec{v}$ represents final velocities.

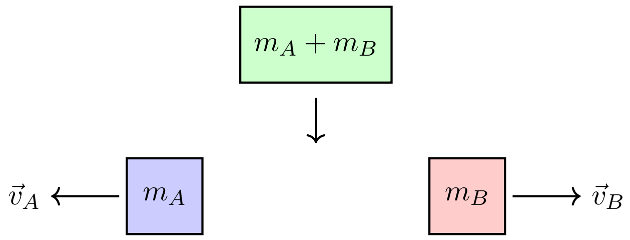
3.2 System Interactions: Internal vs. External Forces

- **Internal Forces:** Forces exerted by objects within the system on each other. These do not change the total momentum (due to Newton's 3rd Law pairs).
- **External Forces:** Forces from outside the system (e.g., friction, gravity). These can change the total momentum.

3.3 Explosions and Recoil

A special case of momentum conservation occurs when a system starts from rest and pushes apart. The initial momentum is zero, so the final total momentum must also be zero.

Before ($p_i = 0$)



After ($m_A\vec{v}_A + m_B\vec{v}_B = 0$)

3.4 Worked Example 3.1: Gun Recoil

Question: A rifle with a mass of 4.0 kg fires a 0.010 kg bullet with a muzzle velocity of 600 m/s. Calculate the recoil velocity of the rifle.

Solution: Let $m_1 = 4.0$ kg (rifle), $m_2 = 0.010$ kg (bullet). Initial velocities $u_1 = 0, u_2 = 0$. Bullet final velocity $v_2 = 600$ m/s.

$$\begin{aligned} 0 &= m_1 v_1 + m_2 v_2 \\ 0 &= (4.0)v_1 + (0.010)(600) \\ -4.0v_1 &= 6.0 \\ v_1 &= -1.5 \text{ m/s} \end{aligned}$$

The recoil velocity of the rifle is 1.5 *m/s in the opposite direction of the bullet.*

4 Collisions in One Dimension

Collisions occur when objects strike each other. We classify collisions based on whether kinetic energy is conserved.

4.1 Elastic Collisions

An **elastic collision** is one in which both momentum and total kinetic energy are conserved. This usually happens at the subatomic level (e.g., gas molecules) or with extremely hard objects.

Elastic Collision Conditions

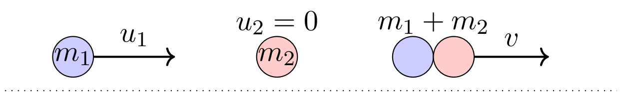
1. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ (Momentum)
2. $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ (Kinetic Energy)

4.2 Inelastic Collisions

In an **inelastic collision**, momentum is conserved, but kinetic energy is **not**. Some kinetic energy is transformed into sound, heat, or the deformation of the objects.

If the objects stick together after the collision, it is called a **perfectly inelastic collision**.

Perfectly Inelastic Collision



$$m_1u_1 = (m_1 + m_2)v$$

4.3 Kinetic Energy Loss

In inelastic collisions, the change in kinetic energy is calculated as:

$$\Delta K = K_f - K_i \tag{6}$$

ΔK will be negative, representing the energy lost to the environment.

4.4 Experimental Lab Analysis: Ticker Tape

In a laboratory setting, we use ticker tape timers or motion sensors to measure the velocities of carts before and after collisions.

- **Elastic cart:** Cart with a spring bumper.
- **Inelastic cart:** Cart with clay or velcro bumpers.

5 Summary and Mastery Exercises

5.1 Key Concepts Review

- **Momentum ($\vec{p} = m\vec{v}$):** A vector quantity representing the motion of a mass.
- **Impulse ($\vec{I} = \vec{F}\Delta t = \Delta\vec{p}$):** The change in momentum caused by a net force.
- **Law of Conservation of Momentum:** $\vec{p}_i = \vec{p}_f$ in an isolated system.
- **Collision Types:** Elastic (KE conserved) vs. Inelastic (KE not conserved).

5.2 Comprehensive Practice Problems

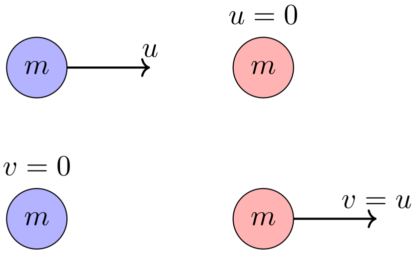
1. **Conceptual:** A car and a heavy truck are moving with the same velocity. Which one has more momentum? If they both need to be stopped in 5 seconds, which one requires more force?
2. **Impulse Calculation:** A tennis ball of mass 0.060 kg moving at 20 m/s is struck by a racket and moves in the opposite direction at 30 m/s. If the contact time is 0.010 s, calculate:
   (a) The change in momentum of the ball.
   (b) The average force exerted by the racket.
3. **Conservation:** A 50 kg ice skater is standing still. He throws a 5 kg medicine ball horizontally at 4 m/s. What is the skater's resulting velocity?
4. **Collision Analysis:** Cart A (2.0 kg) moving at 5.0 m/s hits Cart B (3.0 kg) which is at rest. They stick together after the collision.
   (a) Find the final velocity of the combined carts.
   (b) Calculate the total kinetic energy before and after the collision.
   (c) Is this collision elastic? Explain.

Safety Tip

Always remember that **airbags** don't reduce the change in momentum (Δp) you experience in a crash; they reduce the **force** (F) by making you stop over a longer **time** (Δt). Physics saves lives!

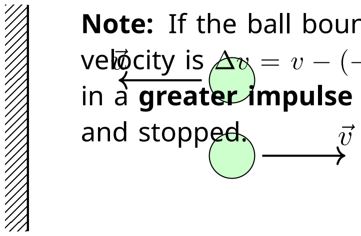
Visual Appendix: Collision Scenarios

Case 1: Elastic Collision (Equal Masses, One at Rest)



The incoming mass stops; the target mass takes all its momentum.

Case 2: The Impulse of a Bounce



Note: If the ball bounces back, the change in velocity is $\Delta v = v - (-u) = v + u$. This results in a **greater impulse** than if the ball simply hit and stopped.

– End of Course Materials –